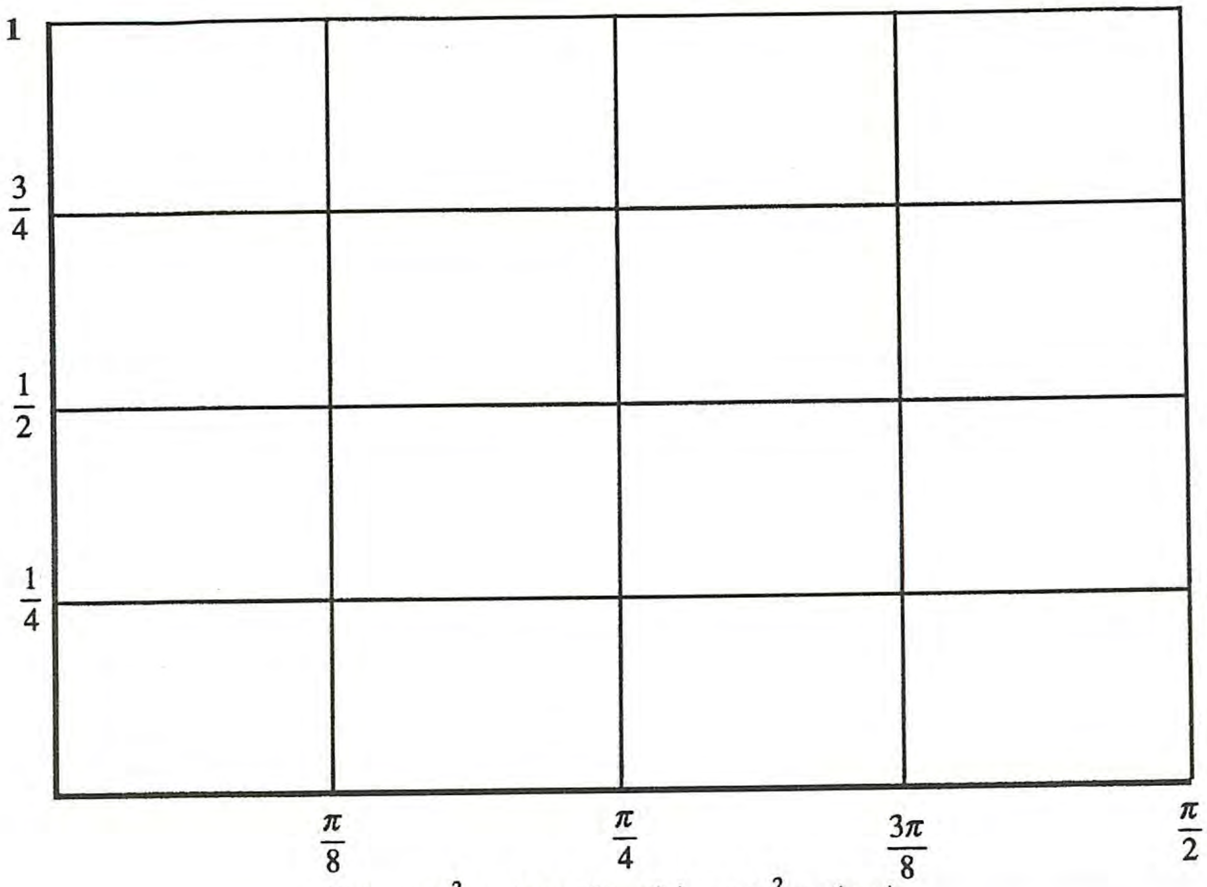
A P Calculus AB
CHAPTER 9
2003
This *is* some new stuff!

		Exam Ch 7 & 8 • • • # 1 R/C Pg 307 - 312 • • • # 2 Pg 312 1 - 27 odd, 28 A 2 / 19	# 3 Pg 312 2 - 26 even 30, 31, 33, 34, 36 • • • # 4 R/C Pg 313 - 318 **No lecture -- just answer questions.** 2 / 20	# 5 Pg . 319 1 - 37 odd, 41 - 45 odd **Discuss HW # 3 and/or have it put on boards after lecture #1. Include pblm 34 pg 312!** **Begin Substitution technique** 2 / 21
# 6 Pg 318-319 2 - 44 even, 46 - 51 **Talk about pblm 41** **Do not collect #11 until #15 assigned** 2/24	# 7 Pg 322 1 - 9, 32, 33 Indicate the formula number you used for each. **Using integral tables. Need to show tables because we generate those terms using "parts" also used in business** 2/25	# 8 Pg 325 - 326 1 - 11, 13, 14, 23, 24 Pblm 6: Change lower bound from zero to 1. Substitute & Evaluate Check using your grapher **Substitution in the definite integral. Show change of limits.** 2 / 26	**Important lecture** Pg 331 - 332 1 - 8, 10 - 17A, 18A, 24, 25 # 9 **Integration by PARTS** **After lecture, discuss hw pblms 23 & 24 fm previous HW** 2 / 27	R/C Pg 332 - 336 # 10 • • • Pg 336 1, 3, 5 - 8, 11, 16, 17 ALSO determine true or false the two equations in the lower right portion of this HW sked. # 11 **Prime 1st, 2nd degree equations** 2 / 28
Important lecture **# 12** Re-Write today's class notes, legibly. Include "comments" which explain the significance of each of the steps.**ALSO** **# 13 Pg 342 16 - 30A** Partial Fractions 3 / 3	# 14 Graph F (x) = Sin^2 x and G (x) = Cos^2 x on the axis provided on back of sked. **Catch up day. Repair past HWs, Finish last night's HW. Have HW #13 put on boards.** **Partial fractions** **Use "put scheme" on yellow paper in NB** 3 / 4	**Important lecture** # 15 Pg 350 1 - 13, 16, 19 **integrals with Sine & Cosine** **"Take out HW Skeds" Look to see if they drew graphs. excel +, so so check or double minus.** **Show verification from pg 350 pblm 1** **coll HW before lecture** 3 / 5	**Important lecture** Pg 354 - 355 1 - 5, 10, 13, 16 # 16 Do problem 13 first using $3x = 2\tan\theta$ then re-do using $u = 4 + 9x^2$ **Trig substitutions** 3 / 6	GQ Pg 356 - 357 1 - 8, 10, 11, 13, 14 # 17 Start working on the review section listed on Monday. 3 / 7
# 18, 19, 20, 21,22 Review Section:. Entire Review section will be done as a unit, complete, in order and correct and turned-in on **the day of the exam. Do *Your* Own Work !** Get help **AFTER** you have made *several* attempts. Pages 357 - 360 : 1 - 12A, 13B, 14A, 17- 20, 27 - 29, 34 - 37, 40 - 43, 44, 46, 47, 49, 53, 148, 151 - 154, 159 3 / 10	3 / 11	3 / 12	**Hint on pblm 11 ...** **let** $u = \cos\theta$ 3 / 13	**Put last year's exam on the chalkboards.** 3 / 14
EXAM Chapter 9 2003 3 / 17			1) $\frac{3}{x-1}+\frac{1}{x-2}\overset{?}{=}\frac{4x-5}{x^2-3x+2}$ 2) $\frac{1}{x-2}-\frac{3}{x^2-4x+4}+\frac{2}{2x-1}\overset{?}{=}\frac{4x^2-19x+12}{2x^3-9x^2+12x-4}$	



$f(x) = \sin^2 x$ (pencil) $g(x) = \cos^2 x$ (pen)

Rewrite each function in terms of sine or cosine but without any exponents. Give a detailed explanation of how to graph each function *without* the aid of any kind of calculator.

REVIEW SECTION ANSWERS AND/OR HINTS

$3\ln|x-1|$

Done

2) $x\ln x - x$; $\frac{1}{2}(\ln x)^2$

4) let $u = e^x$; $3\sec^2\theta = x^2$

6) $\frac{5}{2}x^2 + 3\ln|x+1| + \ln(x^2+1) - \tan^{-1}x$

12a) $-\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta$

$\tan\theta - \theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta$

18) $x + \frac{1}{x-1} + \frac{x-1}{x^2+x+1}$

20) $1 + \frac{1}{x-1} + \frac{2}{x+1} - \frac{1}{x^2+1}$

25) Briefly explain ALL possibilities

29) Page 351 case 3

30) Page 349 eg 8

34) $\frac{4}{3}\sqrt{\sqrt{x}+2}\left(\sqrt{x}+4\right)$

43) $\sin 3x \;\cos 3x = \frac{1}{2}\sin 6x$

46) $\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1}x$

150) So, who needs tables, anyway?

(53) $A(\tau) = \int_1^{\tau} \sqrt{\ln x}\, dx \qquad \frac{dA}{d\tau} = \sqrt{\ln \tau} + c$

(54) $\frac{dy}{dx} = \sin y \qquad \frac{d^2y}{dx^2} = \sin y \cos y$

$$\frac{d^2y}{dx^2} = \cos y \frac{dy}{dt}$$

$$= \cos y \sin y$$

(55)

a) $D_x \int_1^{x^3} e^{\sqrt{\tau}}\, d\tau \Rightarrow \text{let } u = x^3$

$\frac{du}{dx} = 3x^2$

$$D_u \int_1^{u} \left[e^{\sqrt{\tau}}\, d\tau \right] \frac{du}{dx} = e^{\sqrt{u}} \cdot 3x^2 = e^{x^{3/2}} \cdot 3x^2$$

b) $D \int_{x^3}^{2} \sin\sqrt{\tau}\, d\tau = D - \int_2^{x^3} \sin\sqrt{\tau}\, d\tau$

let $u = x^3$

$\frac{du}{dx} = 3x^2 \qquad D_u - \int_2^{u} \sin\sqrt{\tau}\, d\tau \frac{du}{dx} = -\sin\sqrt{u}\; 3x^2 =$

$-3x^2 \sin x^{3/2}$

(56) $D_x \int_{x^2}^{x^3} \sqrt{1+\tau^7}\, d\tau = D \left\{ - \int_a^{x^2} \sqrt{1+\tau^7}\, d\tau + \int_a^{x^3} \sqrt{1+\tau^7}\, d\tau \right\}$

$u = x^3$

$du = 3x^2$

$v = x^2$

$\frac{dv}{dx} = 2x$

$$D \left[\int_a^{u} \sqrt{1+\tau^7}\, d\tau - \int_a^{v} \sqrt{1+\tau^7}\, d\tau \right] = 3x^2\sqrt{1+u^7} - 2x\sqrt{1+u^7} =$$

$$3x^2\sqrt{1+x^{21}} - 2x\sqrt{1+x^{14}}$$

Pg 312

① $\int x^3 dx = \frac{1}{4}x^4$ ② $\int 5x^3 dx = \frac{5}{4}x^4$

③ $\int x^{\frac{1}{3}} dx = \frac{3}{4}x^{\frac{4}{3}}$ ④ $\int \sqrt[3]{x^2}\, dx = \frac{3}{5}x^{5/3}$

⑤ $\int \frac{3\,dx}{\sqrt{x}} = \int 3x^{-\frac{1}{2}} dx = 6x^{\frac{1}{2}}$ ⑥ $\int \frac{dx}{\sqrt[4]{x}} = \int x^{-\frac{1}{4}} dx = \frac{4}{3}x^{3/4}$

⑦ $\int 5e^{-2x} dx = -\frac{5}{2}e^{-2x}$ ⑧ $\int \frac{5}{1+x^2} dx = 5\,\text{Tan}^{-1}x$

⑨ $\int \frac{6\,dx}{x\sqrt{x^2-1}} = 6\,\text{Sec}^{-1}x \quad x \geq 0$ ⑩ $\int \sec^2 3x\, dx = \frac{1}{3}\tan 3x$

⑪ $\int \frac{x^3 dx}{1+x^4} = \frac{1}{4}\ln(1+x^4)$ ⑫ $\int \frac{e^x}{1+e^x} dx = \ln(1+e^x)$

⑬ $\int \frac{\sin x\, dx}{1+\cos x} = -\ln(1+\cos x)$ ⑭ $\int \cos 2x\, dx = \frac{1}{2}\sin 2x$

⑯ $\int \frac{dx}{1+3x} = \frac{1}{3}\ln(1+3x)$ ⑰ $\int \left(3\cos x - \frac{1}{\sqrt{1-x^2}}\right) dx = 3\sin x - \text{Tan}^{-1}x$

⑮ $\int \frac{1+2x}{x+x^2} dx = \ln(x+x^2)$ ⑱ $\int \frac{dx}{x^5} = -\frac{1}{4}x^{-4}$

⑲ $\int (1+3x)x^2 dx = \int (x^2+3x^3) dx = \frac{1}{3}x^3 + \frac{3}{4}x^4$

⑳ $\int \left(\sqrt{x} - \frac{2}{x} + \frac{x}{2}\right) dx = \frac{2}{3}x^{3/2} - 2\ln x + \frac{1}{4}x^2$ $\text{Tan}^{-1}x + \ln(1+x^2)$ $\leftarrow$

㉑ $\int x^2\sqrt{x}\, dx = \int x^{\frac{5}{2}} dx = \frac{2}{7}x^{7/2}$ ㉒ $\int \frac{1+2x}{1+x^2} dx = \int \frac{dx}{1+x^2} + \int \frac{2x\,dx}{1+x^2} =$

㉓ $\int \frac{1+\sqrt{x}}{x} dx = \int \frac{dx}{x} + \int \frac{dx}{\sqrt{x}} = \ln x + 2\sqrt{x}$ $\frac{1}{5}x^5 + 2x^3 + 9x$ $\leftarrow$

㉔ $\int \frac{\sin 2x}{\cos 2x} dx = -\frac{1}{2}\ln(\cos 2x)$ ㉕ $\int (x^2+3)^2 dx = \int x^4 + 6x^2 + 9\, dx$

㉖ $\int (1+e^x)^2 dx = \int (1+2e^x+e^{2x}) dx = x + 2e^x + \frac{1}{2}e^{2x}$

㉗ $\int dx = x$

Pg 312

㉘ $\frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2 \ln(ax+b)\right]$

a)

$$\frac{1}{a^3}\left[a(ax+b) - 2ab + \frac{ab^2}{ax+b}\right] = \frac{1}{a^2}\left(ax+b - 2b + \frac{b^2}{ax+b}\right)$$

$$\frac{1}{a^2}\left(\frac{a^2x^2 + \cancel{2abx} \cancel{+b^2} - \cancel{2abx} - \cancel{2b^2} \cancel{+b^2}}{ax+b}\right) = \frac{x^3}{ax+b}$$

b)

$$-\frac{x^2}{a^2 a\sqrt{ax^2+c}}\,{}^{u} + \frac{2}{a^2}\sqrt{ax^2+c}$$

$$u^{\frac{1}{2}}$$

$$\frac{1}{\cancel{2}} u^{-\frac{1}{2}} \cancel{2}ax$$

$$\frac{-2ax\sqrt{ax^2+c} + \frac{a^2x^3\sqrt{}}{\sqrt{ax^2}}}{a\sqrt{}}$$

P. 307-312

Very complete discussion in text.

Pblms 1-28 appear to be routine, (they are)

(29) $\frac{d^2y}{dx^2} = (2x-1)^5$ Solve for y.

$\frac{dy}{dx} = \int (2x-1)^5 dx = \frac{1}{12}(2x-1)^6 + c_1$

$y = \int \frac{1}{12}(2x-1)^6 + c_1 = \frac{1}{168}(2x-1)^7 + c_1 x + c_2$

(30) $f(x) = \ln x - 9x^3$ $\overset{\text{max}}{\text{min}}$ values $(0,1)$

$f'(x) = \frac{1}{x} - 27x^2 \overset{\text{set}}{=} 0$

$1 - 27x^3 = 0$

$(1-3x)(1+3x+9x^2) = 0$

↑ never zero for $(0,1)$

↑ $x = \frac{1}{3}$ critical value.

using f' test $f'(x)$ for $x < \frac{1}{3}$ $f' > 0$

$f'(x)$ for $x > \frac{1}{3}$ $f' < 0$

a) max at $x = \frac{1}{3}$ ∴ no min value.

max value $\ln(\frac{1}{3}) - 9 \cdot \frac{1}{27} = -\ln 3 - \frac{1}{3}$

b) over closed interval $[\frac{1}{2}, 1]$ since there are no critical pts within the interval max & min must occur at end pts.

$f(\frac{1}{2}) = -\ln 2 - \frac{9}{8}$ $f(1) = -9$ f test

MAX min.

312

(31) $\lim_{x\to 0} \frac{e^{2x}-1-2x}{1-\cos 2x}$ $\frac{0}{0}$ use l'hopital's rule

$\lim_{x\to 0} \frac{2e^{2x}-2}{2\sin 2x} = \lim_{x\to 0} \frac{e^{2x}-1}{\sin 2x}$ still $\frac{0}{0}$

$\lim_{x\to 0} \frac{2e^{2x}}{2\cos 2x} = \frac{2}{2} = 1$

(32) $y = \sin^2 x$ $[0, \pi]$ hint $\sin^2 x = \frac{1-\cos 2x}{2}$

$\frac{1}{2}\int_0^{\pi} 1 - \cos 2x\,dx = \int_0^{\pi} \frac{1}{2}\,dx - \int_0^{\pi} \frac{1}{2}\cos 2x\,dx$

$\frac{1}{2}x\Big|_0^{\pi} + \sin 2x\Big|_0^{\pi} = \frac{\pi}{2} + 0 = \frac{\pi}{2}$

(33) $\int \frac{x\,dx}{1+x^4} = \frac{1}{2}\tan^{-1} x^2$

(34) Jack: $\int 2\cos\theta\sin\theta\,d\theta = -\cos^2\theta$

Jill: $\int 2\cos\theta\sin\theta\,d\theta = \sin^2\theta$

Take der of Both answers Jack' $= +2\cos\theta\sin\theta$

Jill' $= 2\sin\theta\cos\theta$.

Both are the same! They should be. They differ only in a constant ie $1-\cos^2\theta = \sin^2\theta$

Jack:

(35) $\int \sec\theta\, d\theta = \ln(\sec\theta + \tan\theta)$

Jill: $\int \sec\theta\, d\theta = \ln\left(\tan\frac{\theta}{2} + \frac{\pi}{4}\right)$

Jack' $= \dfrac{\sec\theta\tan\theta + \sec^2\theta}{\sec\theta + \tan\theta} = \sec\theta$

Jill's $= \dfrac{\frac{1}{2}\sec^2\frac{\theta}{2}}{\tan\frac{\theta}{2} + \frac{\pi}{4}} \overset{u}{\underset{v}{=}} \left(\tan\frac{\theta}{2} + \frac{\pi}{4}\right)\left(\sec\frac{\theta}{2}\right)\frac{1}{2}\sec\frac{\theta}{2}\tan\frac{\theta}{2}$

Jill's uses $\frac{1}{2}$ angles

(36) a) eg $\int f(x)\,dx \int g(x)\,dx \neq \int f(x)\,g(x)\,dx$

$\int \frac{1}{x}\,dx \int x\,dx \neq \int dx$

$\ln x + \frac{x^2}{2} \neq x$

b) given $\int f(x)\,g(x)\,dx = f(x)\int g(x)\,dx$.

take derivative (using FTC (2nd)

$$\frac{d\left(\int f(x)\,g(x)\,dx\right)}{dx} = f(x)\,g(x)$$

$$d\Big(\overset{u}{f(x)}\overset{v}{\int g(x)\,dx}\Big) = f'(x)\int g(x)\,dx + f(x)g(x)$$

So

$$f(x)\,g(x) = f'(x)\int g(x)\,dx + f(x)\,g(x)$$

$$0 = f'(x)\int g(x)\,dx$$

Since g is not constant only choice for product to be zero is $f'(x)$ & if $f(x)$ is NOT constant it is Not true. ∴ f must be constant.

~~we could~~ divide bottom into top

1st Lecture

$$\int \frac{x^2+1}{2x-3}\,dx$$

"Improper fraction" ↑

$$\begin{array}{r} \frac{1}{2}x + \frac{3}{4} \\ 2x-3 \overline{)\, x^2+1 \quad} \\ x^2 - \frac{3}{2}x \\ \hline \frac{3}{2}x \\ \frac{3}{2}x - \frac{9}{4} \\ \hline \frac{13}{4} \end{array}$$

$$\frac{13}{4}\int \frac{dx}{2x-3}$$

$$\frac{13}{4}\cdot\frac{1}{2}\ln|2x-3|$$

$$\int \frac{1}{2}x + \frac{3}{4} + \frac{13}{4(2x-3)}\,dx$$

$$\frac{x^2}{4} + \frac{3x}{4} + \frac{13}{8}\ln|2x-3| + c$$

OR

$$\int \frac{x^2+1}{2x-3}\,dx$$

~~let $u = 2x-3$~~ $du = \frac{1}{2}\,dx$

~~$x = \frac{1}{2}(u+3)$~~ $2\,du = dx$

~~$x^2 = \frac{(u+3)^2}{4}$~~

~~$\int \left(\frac{(u+3)^2}{4} + 1\right) = \int \frac{u^2}{4} + \frac{\overset{3}{6}u}{24} + \frac{13}{4}\,du$~~

~~$\frac{u^3}{12} + \frac{3u^2}{4} + \frac{13}{4}u$~~

"Notice the der of the inside".

$\int \sin(\underline{x^2})\,\underline{2x\,dx}$ $\quad \int \sin u\,du \quad -\cos u = -\cos x^2$

let $u = x^5$

$\int e^{\underline{x^5}} \cdot \underline{5x^4\,dx} = \int e^u\,du = e^u = e^{x^5}$

$\int \underline{\sin^2\theta}\,\underline{\cos\theta\,d\theta}$

$$\int (1-x^3)^5 x^2\,dx$$

let $u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$$\frac{1}{3}\int (1-u)^5\,du$$

$$\frac{1}{3}\;\frac{(1-u)^6}{6}\underset{\substack{\uparrow\\ \text{der of}\\ \text{inside.}}}{(-1)} + C$$

$$-\frac{1}{18}(1-x^3)^6 + C$$

let $u = 1 - x^3$

$du = -3x^2\,dx$

$-\frac{1}{3}du = x^2\,dx$

$$-\frac{1}{3}\int u^5\,du$$

$$-\frac{1}{3}\;\frac{u^6}{6} + c$$

$$\frac{-(1-x^3)^6}{18} + c$$

This method is not ↗ This
as reliable as
method. because of
requirement to remember
"der of inside".

open lecture w/ : find deriv of $\frac{1}{18}(1+x^3)^6$. this sets up ~~Eg~~ Eg 4 show eg 4 then go to eg 6.

Pg 313 – 318

Lecture.

Pg 314

eg 1–5 routine.

If I chose to let $u = x^2+1$
$du = 2x\,dx$
But I need $\frac{dx}{2x-\ldots}$

eg 6 $\displaystyle\int \frac{x^2+1}{2x-3}\,dx$ Try by: Let $u = 2x-3$

$du = 2\,dx$ and $x = \frac{u+3}{2}$

$$\int \frac{\left(\frac{u+3}{2}\right)^2+1}{u}\left(\frac{1}{2}du\right) = \frac{1}{2}\cdot\frac{1}{4}\int \frac{u^2+6u+9+4}{u}\,du$$

$$\frac{1}{8}\int u + 6 + \frac{13}{u}\,du = \frac{1}{8}\left(\frac{u^2}{2} + 6u + 13\ln|u|\right)$$

$$\frac{(2x+3)^2}{16} + \frac{3(2x-3)}{4} + \frac{13}{8}\ln|2x-3|$$

eg 7 $\displaystyle\int \sqrt{1-\frac{4}{5}\sin^2\theta}\,d\theta$

Since we do not have a $\cos\theta\,d\theta$ it will not be easy. we must try something so seeing nothing more promising we try $u = \sin\theta$ anyway.

YUK

Just point out what "a mess" looks like!

Let $u = \sin\theta$
$du = \cos\theta\,d\theta$

$$\int \sqrt{1-\frac{4}{5}u}\;\frac{du}{\cos\theta} =$$

$$\int \sqrt{1-\frac{4}{5}u}\;\frac{du}{\sqrt{1-u^2}}$$

Still a mess, we needed the one to be a $\frac{4}{5}$ or a $\cos\theta$ on the out side.

So having exhausted all obvious paths we give up & hope it does not have an elementary fctn.

Read pgs 317–318 carefully.

Pg 318

① $\int (1+3x)^5 \, 3dx \quad u = 1+3x \quad du = 3dx$

$\int u^5 du = \frac{1}{6}u^6 \Rightarrow \frac{1}{6}(1+3x)^6$

② $\int \frac{x\,dx}{(1+x^2)^3} \quad u = 1+x^2 \quad du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u^3} = -\frac{1}{4}u^{-2} \Rightarrow -\frac{1}{4(1+x^2)^2}$

③ $\int e^{\sin\theta}\cos\theta\,d\theta \quad u = \sin\theta \quad du = \cos\theta\,d\theta$

$\int e^u du = e^u \Rightarrow e^{\sin\theta}$

④ $\frac{1}{2}\int \frac{2x\,dx}{\sqrt{1+x^2}} \quad u = 1+x^2 \quad du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{du}{u^{\frac{1}{2}}} = u^{\frac{1}{2}} \Rightarrow \sqrt{1+x^2}$

⑤ $\int \sqrt{1+x^2}\,x\,dx \quad u = 1+x^2 \quad du = 2x\,dx$

$\frac{1}{2}\int \sqrt{1+x^2}\,2x\,dx = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{3}u^{3/2} = \frac{1}{3}(1+x^2)^{3/2}$

⑥ $\int \sin 2x\,dx \quad u = 2x \quad du = 2dx$

$\frac{1}{2}\int \sin 2x\,2dx = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u \Rightarrow -\frac{1}{2}\cos 2x,$

⑦ $\int \frac{e^{2x}dx}{(1+e^{2x})^2} \quad u = 1+e^{2x} \quad du = 2e^{2x}dx$

$\frac{1}{2}\int \frac{2e^{2x}dx}{(1+e^{2x})^2} = \frac{1}{2}\int \frac{du}{u^2} = -\frac{1}{2}u^{-1} = \frac{-1}{2(1+e^{2x})}$

⑧ $\int e^{3x}dx \quad u=3x \quad du=3dx$

$\frac{1}{3}\int e^{u}du = \frac{1}{3}e^{u} \Rightarrow \frac{1}{3}e^{3x}$

⑨ $\int \frac{e^{\frac{1}{x}}dx}{x^2} \quad u=\frac{1}{x} \quad du=-\frac{1}{x^2}dx$

$-\int e^{u}du = -e^{u} \Rightarrow -e^{\frac{1}{x}}$

⑩ $\int \frac{dx}{\sqrt{1-9x^2}} \quad u=3x \quad du=3dx$

$\frac{1}{3}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{3}\operatorname{Sin}^{-1}u \Rightarrow \frac{1}{3}\operatorname{Sin}^{-1}3x$

⑪ $\int \frac{t\,dt}{\sqrt{2-5t^2}} \quad u=2-5t^2 \quad du=-10t\,dt$

$-\frac{1}{10}\int \frac{du}{u^{\frac{1}{2}}} = -\frac{2}{10}u^{\frac{1}{2}} \Rightarrow -\frac{1}{5}\sqrt{2-5t^2}$

⑫ $\int \tan\theta\sec^2\theta\,d\theta \quad u=\tan\theta \quad du=\sec^2\theta\,d\theta$

$\int u\,du = \frac{1}{2}u^2 \Rightarrow \frac{1}{2}\tan^2\theta$

⑬ $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx \quad u=\sqrt{x} \quad du=\frac{1}{2}x^{-\frac{1}{2}}dx$

$2du = x^{-\frac{1}{2}}x$

$\int \sin u\,du = -\cos u \Rightarrow -2\cos\sqrt{x}$

⑭ $\int \frac{(\ln x)^4}{x}dx \quad u=\ln x \quad du=\frac{1}{x}dx$

$\int u^4du = \frac{1}{5}u^5 \Rightarrow \frac{1}{5}(\ln x)^5$

(15) $\int \frac{\sin(\ln x)}{x}\,dx \quad u = \ln x \quad du = \frac{1}{x}\,dx$

$\int \sin u\,du = -\cos u \Rightarrow -\cos(\ln x)$

(16) $\int (1-x^2)^5\,x\,dx$ Let $u = 1-x^2 \quad du = -2x\,dx$

$-\frac{1}{2}\int u^5\,du = -\frac{1}{12}u^6 \Rightarrow -\frac{1}{12}(1-x^2)^6$

(17) $\int \frac{x\,dx}{(x^2+1)^3}$ Let $u = x^2+1 \quad du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u^3} = -\frac{1}{2}\cdot\frac{1}{2}u^{-2} = -\frac{1}{4}u^{-2} \Rightarrow \frac{-1}{4(x^2+1)^2}$

(18) $\int \sqrt[3]{1+x^2}\,x\,dx$ Let $u = 1+x^2 \quad du = 2x\,dx$

$\frac{1}{2}\int u^{\frac{1}{3}}\,du = \frac{1}{2}\cdot\frac{3}{4}u^{\frac{4}{3}} \Rightarrow \frac{3}{8}(1+x^2)^{\frac{4}{3}}$

Book ANS wrong,

(19) $\int \frac{\sin\theta\,d\theta}{\cos^2\theta}$ Let $\underline{u = \cos\theta} \quad du = -\sin\theta\,d\theta$

$-\int \frac{du}{u^2} = \frac{1}{u} \Rightarrow \frac{1}{\cos\theta} = \sec\theta.$

(20) $\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt$ Let $u = \sqrt{t} \quad 2du = \frac{dt}{\sqrt{t}}$

$2\int e^u\,du = 2e^u \Rightarrow 2e^{\sqrt{t}}$

(21) $\int \sin 3\theta\,d\theta \quad u = 3\theta \quad du = 3\,d\theta.$

$\frac{1}{3}\int \sin u\,du = -\frac{1}{3}\cos u \Rightarrow -\frac{1}{3}\cos 3\theta.$

318

㉒ $\int e^x \sin e^x \, dx$ let $e^x = u$ $du = e^x \, dx$

$\int \sin u \, du = -\cos u \Rightarrow -\cos e^x$

㉓ $\int \frac{dx}{\sqrt{2x+5}}$ let $u = 2x+5$ $du = 2\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u} \Rightarrow \sqrt{2x+5}$

㉔ $\int (x-3)^{5/2} dx$ let $u = x-3$ $du = dx$

$\int u^{5/2} du = \frac{2}{7} u^{7/2} \Rightarrow \frac{2}{7}(x-3)^{7/2}$

㉕ $\int \frac{dx}{(4x+3)^3}$ let $u = 4x+3$ $du = 4\,dx$

$\frac{1}{4}\int \frac{du}{u^3} = -\frac{1}{8} u^{-2} = \frac{-1}{8(4x+3)^2}$

㉖ $\int \frac{(2x+3)dx}{x^2+3x+2}$ let $u = x^2+3x+2$
$du = 2x+3 \ dx$

$\int \frac{du}{u} = \ln|u| \Rightarrow \ln|x^2+3x+2|$

㉗ $\int \frac{(2x+3)\,dx}{(x^2+3x+5)^4}$ let $u = x^2+3x+5$
$du = 2x+3 \ dx$

$\int \frac{du}{u^4} = -\frac{1}{3} u^{-3} \Rightarrow \frac{-1}{3(x^2+3x+5)^3}$

㉘ $\int e^{2x} dx$ let $u = e^{2x}$
$\frac{1}{2}$ $du = 2e^{2x} dx$

$\frac{1}{2}\int du = \frac{1}{2} e^{2x} \Rightarrow \frac{1}{2} e^{2x}$

Pg 318

(29) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^3}$ let $u = \sqrt{x}$
$2du = \frac{dx}{\sqrt{x}}$

$\int \frac{2du}{u^3} = (2)-\frac{1}{2}u^{-2} = \frac{-\cancel{2}}{\cancel{2}(1+\sqrt{x})^2} = \frac{-1}{(1+\sqrt{x})^2}$

(30) $\int x^4 \sin x^5 dx$ let $u = x^5$
$du = 5x^4 dx$

$\frac{1}{5}\int \sin u\, du = -\frac{1}{5}\cos u \Rightarrow -\frac{1}{5}\cos x^5$

(31) $\int \frac{dx}{e^x} = -e^{-x}$

(32) $\int (3+x^2)^2 dx = \int 9 + 6x^2 + x^4\, dx = 9x + 2x^3 + \frac{1}{5}x^5$

(33) $\int (2e^x + 4e^{-x})^2 dx = \int (4e^{2x} + 16 + 16e^{-2x})dx =$

$2e^{2x} + 16x - 8e^{-2x}$

(34) $\int \frac{x+3x^2}{\sqrt{x}} dx = \int \sqrt{x} + 3x^{3/2} dx = \frac{2x^{3/2}}{3} + \frac{6x^{5/2}}{5}$

(35) $\int \frac{x}{1+x^4} dx$ let $u = x^2$ $du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1} u \Rightarrow \frac{1}{2}\mathrm{Tan}^{-1} x^2$

(36) $\int \frac{x^3 dx}{1+x^4}$ let $u = 1+x^4$
$du = 4x^3 dx$

$\frac{1}{4}\int \frac{du}{1+u} = \frac{1}{4}\ln|1+u| = \frac{1}{4}\ln(1+x^4)$

(37) $\int \frac{x\,dx}{\sqrt{1-x^4}}$ let $u = x^2$ $du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\mathrm{Sin}^{-1} u = \frac{1}{2}\mathrm{Sin}^{-1} x^2$

Pg 319

$$x+1\,\overline{)\,x}\quad 1$$
$$\frac{x+1}{-1}$$

(38) $\int \frac{x\,dx}{1+x} =$

$\int dx + \int \frac{dx}{x+1} = x - \ln|x+1|$

Pg 312 #32
~~Pg 448~~

$\frac{1}{2}x - \frac{1}{12}\sin 6x$

$\frac{1}{2} - \frac{1}{2}\cos 6x$

$\frac{1}{2}(1-\cos^2 6x)$

(39) $\int \sin^2 3x\,dx$ $\quad u = 3x$, $du = 3dx$

$\frac{1}{3}\int \sin^2 u\,du$

not elementary
Book answer wrong →

(40) $\int \frac{e^x\,dx}{1+e^{2x}}$ $\quad$ let $u = e^x$, $du = e^x\,dx$

$\int \frac{dy}{1+u^2} = \tan^{-1} u = \tan^{-1} e^x$

(41) $\int \sqrt{3+x^2}\;x^3\,dx$

$\int \sqrt{\cancel{3x^6+x^8}}\;dx$

$\int \frac{x}{5}(3+x^2)^{\frac{1}{2}}\;5x^2\,dx$

let $u = 5\left[\overset{u}{x}(\overset{v}{3+x^2}) - 2x^2 - 3x\right]$

$du = 5(3+x^2+2x-2x-3)$

$5*x^2\,dx$

$\int 3x^2+x^4\;\;x^2\,dx$

$6x+4x^3$

$2x(3+2x)$

$\frac{x^3}{15} - \frac{2x}{5}\quad \frac{1}{5}(\overset{u}{x^2-2})(\overset{v}{3+x^2})^{3/2}$

$\frac{2x}{5}(3+x^2)^{3/2} + \frac{1}{5}\cdot\frac{3}{\cancel{2}}\cdot\cancel{2}x\,(x^2-2)(3+x^2)^{1/2}$

$\frac{x}{5}(3+x^2)^{1/2}\left(2(3+x^2)+3x^2-6x\right)$

$\frac{x}{5}(3+x^2)^{1/2}\left(6+2x^2+3x^2-2x\right)$

$\frac{x}{5}(3*x^2)^{1/2}(5x^2) = (3+x^2)^{\frac{1}{2}}x^3$

$\int \sqrt{3+x^2}\;x^2\,x\,dx$ $\quad u^2 = x^2+3$ $\quad \cancel{2}u\,du = \cancel{2}x\,dx$

$u = \sqrt{x^2+3}$

$\int u(u^2-3)\,u\,du$

$\int u^4 - 3u^2\,du$

$\frac{u^5}{5} - u^3 \Big| = \frac{(x^2+3)^{5/2}}{5} - (x^2+3)^{3/2}$

we need a $\sin 3\theta$.

(42) $\int \sqrt{1+\cos 3\theta}\, d\theta \; \frac{\sqrt{1-\cos 3\theta}}{\sqrt{1-\cos 3\theta}} = \int \frac{\sin 3\theta}{\sqrt{1-\cos 3\theta}}\, d\theta$

let $u = 1 - \cos 3\theta$

$du = 3 \sin 3\theta$

$\frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{2}{3}\sqrt{u} \Rightarrow \frac{2}{3}\sqrt{1-\cos 3\theta}$

(43) $\int (e^{2x}+1) e^{-x}\,dx = \int (e^{x}+e^{-x})\,dx = e^{x} - e^{-x}$

(44) $\int \frac{dx}{\sqrt{1-5x^2}} = \int \frac{dx}{\sqrt{1-(\sqrt{5}x)^2}} = \frac{\sqrt{5}}{5} \sin^{-1} x\sqrt{5}$

(45) $\int \frac{dx}{1+4x^2}$ let $u = 2x$

$du = 2\,dx$

$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1} u \Rightarrow \frac{1}{2}\tan^{-1} 2x$

(46) $\int x \cos x\, dx$ MANufacture a F (we did This Pblm 16 pg 287

$x \sin x + \cos x$

$x \cos x + \sin x - \sin x$. $'$,

$\int x \cos x\, dx = x \sin x + \cos x$

(47) $\int \frac{\cos x}{x}\, dx$ Not elementary

(48) $\int \frac{x\, dx}{\ln x}$ let $u = \cancel{\ln x}$ Not elementary

$du = \cancel{\frac{1}{x}}\, dx$

(49)

$\int \frac{\ln x^2}{x}\, dx = 2\int \frac{\ln x}{x}\, dx$ let $u = \ln x$

$du = \frac{1}{x}\, dx$

$2\int u\, du = u^2 = (\ln x)^2$

(50) $\int \sqrt{x-1}\,\sqrt{x}\,\sqrt{x+1}\, dx$ Not elementary

(51) $\int \sqrt{x-1}\,\sqrt{x+1}\; x\, dx = \int \sqrt{x^2-1}\; x\, dx$ let $u = x^2-1$

$du = 2x\, dx$

$\frac{1}{2}\int u^{\frac{1}{2}}\, du = \frac{1}{2}\frac{2}{3} u^{3/2} = \frac{1}{3}(x^2-1)^{3/2}$

using $\int$ tables. Pg 319–322

eg 1 $\int \sin^6 2x\,dx$

turn to pg 948 formula 46

$$\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax\,dx$$

So since $n = 6$ and $a = 2$ we get.

$$-\frac{\sin^5 2x \cos 2x}{12} + \frac{5}{6}\int \sin^4 2x\,dx$$

using formula 46 again where $n = 4$ $a = 2$

$$-\frac{\sin^5 2x \cos 2x}{12} + \frac{5}{6}\left[-\frac{\sin^2 2x \cos 2x}{8} + \underbrace{\frac{3}{4}\int \sin^2 2x\,dx}_{\text{using formula 44}}\right]$$

$$-\frac{\sin^5 2x \cos 2x}{12} - \frac{5(\sin^2 2x \cos 2x)}{48} + \frac{3}{4}\left[\frac{x}{2} - \frac{\sin 4x}{8}\right]$$

eg 2 a) $\int \frac{dx}{x^2-5x+3}$ pg 948 #35 $25 > 12$

$$\frac{1}{\sqrt{13}} \ln\left(\frac{2x-5-\sqrt{13}}{2x-5+\sqrt{13}}\right)$$

b) #36

c) #37

eg 3 $\int \sqrt{\sin^2\theta + 5}\,\cos\theta\,d\theta$ use substitution

Let $u = \sin\theta$
$du = \cos\theta\,d\theta$

$\int \sqrt{u^2+5}\,du = \cancel{\frac{2}{3}(u^2+5)^{3/2}\,2u}$

oops Jumped Too Soon — use tables now.

↓

p. 319-322
Lecture

$\int \sqrt{u^2+5}\, du$ this is in the form (see pg 319) $x^2 \pm p^2$ where $p^2 = 5$ "Group 2"

pg 948 #31

$$\int \sqrt{x^2 \pm p^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm p^2} \pm p^2 \ln\left|x + \sqrt{x^2 \pm p^2}\right|\right]$$

So

$$\int \sqrt{u^2+5}\, du = \frac{1}{2}\left[u\sqrt{u^2+5} + 5 \ln\left|u + \sqrt{u^2+5}\right|\right]$$

but $u = \sin^2\theta$ so

$$\int \sqrt{\sin^2\theta + 5}\, \cos\theta\, d\theta =$$

$$\frac{1}{2}\left[\sin\theta\sqrt{\sin^2\theta+5} + 5 \ln\left|\sin\theta + \sqrt{\sin^2\theta+5}\right|\right]$$

$\boxed{\text{eg 4} \int \frac{x^5\, dx}{\sqrt{x^3+1}}}$ if $u = x^3$, $du = 3x^2$ so $x^5 = \frac{1}{3} u\, du$

$$\int \frac{\frac{1}{3} u\, du}{\sqrt{u+1}} = \frac{1}{3}\int \frac{u\, du}{\sqrt{u+1}}$$ Not in our Book.

from CRC Book.

$$\int \frac{x\, dx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx}$$ where $a = 1$, $b = 1$

$$= -\frac{2}{3}\left[\frac{2-u}{3}\sqrt{1+u}\right]$$

since $x^5 = x^3 \cdot x^2$, $u = x^3 - 1$
then $(u-1) = x^3$
$\frac{1}{3} du = x^2$ ∴ $x^5 = \frac{1}{3}(u-1)\, du$

w/ different Substitution Try again: let $u = x^3+1$
$du = 3x^2$

$$\frac{1}{3}\frac{1}{3}\int \frac{(u-1)\, du}{\sqrt{u}} = \frac{1}{9}\int \sqrt{u} - \frac{1}{\sqrt{u}}\, du =$$

$$\frac{1}{9}\left[\frac{2}{3}u^{3/2} - 2u^{\frac{1}{2}}\right] = \frac{2}{9}\left[\frac{2u^{\frac{1}{2}}}{3}(u-2)\right] \Rightarrow \frac{2}{9}\left[\frac{2}{3}(x^3-2)\, x^{3/2}\right]$$

Pg 322-323
HW.

① $\int (3x+5)^5\,dx$

#11 $\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1}$ $n = 5,\ a = 3,\ b = 5$

$\frac{1}{18}(3x+5)^6$

#11 ② $\int \frac{dx}{(3x+7)^3} = \frac{-1}{6}(3x+7)^{-2}$ $n = -3$ $a = 3$ $b = 7$

#14 ③ $\int \frac{x\,dx}{(2x-5)^2} =$ $a = 2$ $b = -5$

$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$

$= \frac{-5}{4(2x-5)} + \frac{1}{4}\ln|2x-5|$

#16 ④ $\int \frac{dx}{x^2(3x-2)}$ $\int \frac{dx}{x^2(ax+b)} = \frac{-1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$

$a = 3$ $b = -2$

$= \frac{1}{2x} + \frac{3}{4}\ln\left|\frac{3x-2}{x}\right|$

#17 ⑤ $\int \sqrt{5x-7}\,dx$ $\int \sqrt{ax+b}\,dx = \frac{2}{3a}\sqrt{(ax+b)^3}$

$a = 5$ $b = -7$

$\frac{2}{15}\sqrt{(5x-7)^3}$

#18 ⑥ $\int x\sqrt{2x+3}\,dx$ $\int x\sqrt{ax+b}\,dx =$

$a = 2$ $b = 3$ $\frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$

$\frac{2(6x-6)}{60}\sqrt{2x+3} = \frac{(x-1)}{5}\sqrt{2x+3}$

Pg 323

㉜ $\int \sin 3x \cos 2x \, dx$ $a=3 \quad b=2$

#55 $\int \sin ax \cos bx \, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$

$$= -\frac{\cos x}{2} - \frac{\cos 5x}{10}$$

㉟ $\int x^2 \cos 3x \, dx$ $n=2 \quad a=3$

#54 $\int x^n \cos ax \, dx = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$

$$= \frac{1}{3} x^2 \sin 3x - \frac{2}{3} \int x \sin 3x \, dx$$ #51

$$-\frac{2}{3}\left[\frac{1}{9} \sin 3x - \frac{1}{3} x \cos 3x\right]$$

$$= \frac{1}{3} x^2 \sin 3x - \frac{2}{27} \sin 3x + \frac{2}{9} x \cos 3x,$$

㊻ $\int \frac{x^5 \, dx}{(5x^3+3)^2}$ $u = x^3$
$du = 3x^2 \, dx$

#14 $\frac{1}{3} \int \frac{u \, du}{(5u+3)^2}$ $\int \frac{x \, dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln(ax+b)$

$a=5 \quad b=3$

$$\frac{1}{3}\left[\frac{3}{25(5u+3)} + \frac{1}{9} \ln|5u+3|\right]$$

$$\frac{1}{25(5x^3+3} + \frac{1}{27} \ln|5x^3+3|$$

Pg 323

㊼ $\int x^2\sqrt{5x^3+2}\,dx$ Let $u = 5x^3+2$

$du = 15x^2\,dx$

#17

$\frac{1}{15}\int \sqrt{u+2}\,du$ or set up like next pblm.

㊽ $\int \frac{x\,dx}{\sqrt{2x^2+3}}$ Let $u = 2x^2+3$

$du = 4x\,dx$

$\frac{1}{4}\int \frac{du}{\sqrt{u}}$ #20 or do w/out table.

$x(x-1)(x+1)$

1996

$$\int_{-1}^{2} |x^3 - x|\, dx$$

$- \;0\; + \;0\; - \;0\; +$

$-1 \quad 0 \quad 1 \quad 2$

$$\int_{-1}^{0} x^3 - x\, dx + \int_{0}^{1} x - x^3\, dx + \int_{1}^{2} x^3 - x\, dx$$

$$\int_{4}^{9} \frac{\sqrt{x}\, dx}{\left(30 - x^{\frac{3}{2}}\right)^2}$$

Notice denom is not zero in the interval.

find answer $\frac{19}{99}$

let $u = 30 - x^{\frac{3}{2}}$

$du = -\frac{3}{2} x^{\frac{1}{2}} dx$

$-\frac{2}{3} du = x^{\frac{1}{2}} dx$

$x = 4 \quad u = 22$

$x = 9 \quad u = 3$

Substitution into definite integral Pg 323-325
Lecture.

Read Pgs 323-325 HW Pg 325-326 1-24

All

eg 1 $\int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta\,d\theta$ using substitution we must also change limits of integration

Let $u = \sin\theta$
$du = \cos\theta\, d\theta$.

So far we have $\int u^2\,du$

Now the limits of integration must be adjusted to u from θ.

when $\theta = 0$ $\sin\theta = 0$ $\therefore u = 0$
$\theta = \frac{\pi}{2}$ $\sin\theta = 1$ $\therefore u = 1$ So

$$\int_0^1 u^2\,du = \frac{1}{3}u^3\Big|_0^1 = \frac{1}{3}$$ thus.

$$\int_0^{\frac{\pi}{2}} \sin^2\theta\cos\theta\,d\theta = \frac{1}{3}$$

eg 2 $\int_1^5 \sqrt{1+3x}\,dx$ Let $1+3x = u$
$du = 3\,dx$

$x=1 \Rightarrow u=4$
$x=5 \Rightarrow u=16$ So $\frac{1}{3}\int_4^{16} u^{\frac{1}{2}}\,du =$

$$\frac{2}{3}u^{3/2}\Big|_4^{14} = \frac{1}{3}\,\frac{2}{3}\left[16^{3/2} - 4^{3/2}\right] = \frac{2}{9}\left[64 - 8\right] =$$

$$\frac{2}{9}\left[56\right] = \frac{112}{9}$$

end of lecture

Pg 325

① $\int_0^1 (e^x+1)^3 e^x\,dx$ let $u = e^x+1$ $x=0 \Rightarrow u=2$

$du = e^x$ $x=1 \Rightarrow u=e+1$

$$\int_2^{e+1} u^4\,du = \frac{1}{4}\left[u^4\Big|_2^{e+1}\right] = \underline{\frac{1}{4}\left((e+1)^4-16\right)}$$

② $\int_0^1 (x^4+1)^5 x^3\,dx$ let $u = x^4+1$ $x=0 \Rightarrow u=1$

$du = 4x^3\,dx$ $x=1 \Rightarrow u=2$

$$\frac{1}{4}\int_1^2 u^5\,du = \frac{1}{24}u^6\Big|_1^2 = \frac{2^6}{2^3\cdot 3} - \frac{1}{24} = \frac{63}{24} = \frac{21}{8}$$

③ $\int_0^{\frac{\pi}{4}} \sin 2\theta\,d\theta$ let $2\theta = u$ $\theta=0 \Rightarrow u=0$

$du = 2\,d\theta$ $\theta=\frac{\pi}{4} \Rightarrow u=\frac{\pi}{2}$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \sin u\,du = -\frac{1}{2}\cos u\Big|_0^{\frac{\pi}{2}} = 0+\frac{1}{2}$$

④ $\int_0^{\frac{\pi}{2}} \frac{\cos\theta\,d\theta}{\sqrt{1+\sin\theta}}$ let $1+\sin\theta = u$ $\theta=0 \Rightarrow u=1$

$du = \cos\theta\,d\theta$ $\theta=\frac{\pi}{2} \Rightarrow u=2$

$$\int_1^2 \frac{du}{\sqrt{u}} = 2\sqrt{u}\Big|_1^2 = 2\sqrt{2}-2$$

⑤ $\int_1^2 \frac{x^2\,dx}{2x^3+4}$ let $u = 2x^3+4$ $x=1 \Rightarrow u=6$

$du = 6x^2$ $x=2 \Rightarrow u=20$

$$\frac{1}{6}\int_6^{20} \frac{du}{u} = \frac{1}{6}\ln|u|\Big|_6^{20} = \frac{1}{6}\left[\ln 20 - \ln 6\right] =$$

$$\frac{1}{6}\ln\left(\frac{10}{3}\right)$$

Pg 325

⑥ $\int_1^e \frac{\sqrt{\ln x}}{x}\,dx$ let $u = \ln x$ $x = 1 \Rightarrow u = 0.$

$du = \frac{1}{x}\,dx$ $x = e \Rightarrow u = 1$

change this limit to 1 fm 0.

$\int_0^{\overset{1}{e}} u^{\frac{1}{2}}\,du = \frac{2}{3}u^{3/2}\Big|_0^1 = \frac{2}{3}$

$\left(\sqrt{\ln x}\right)$ $\frac{1}{2x\sqrt{\ln x}}$

$\frac{2}{3}(\ln x)^{3/2}\Big|_1^e =$

$\frac{2}{3}\frac{3}{2}\frac{(\ln x)^{1/2}}{x}\Big|_1 =$

⑦ $\int_2^3 x\sqrt{x^2-4}\,dx =$ let $u = x^2-4$ $x=2 \Rightarrow u=0$

$du = 2x\,dx$ $x=3 \Rightarrow u=5$

$\frac{1}{2}\int_0^5 u^{\frac{1}{2}}\,du = \frac{1}{\cancel{2}}\frac{\cancel{2}}{3}u^{3/2}\Big|_0^5 = \frac{1}{3}\left(5^{3/2}\right)$

⑧ $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^3 2\theta \cos 2\theta\,d\theta$ let $u = \sin 2\theta$ $\theta = \frac{\pi}{6} \Rightarrow u = \frac{\sqrt{3}}{2}$

$du = 2\cos 2\theta\,d\theta$ $\theta = \frac{\pi}{4} \Rightarrow u = 1$

$\frac{1}{2}\int_1^{\frac{\sqrt{3}}{2}} u^3\,du = \frac{1}{8}u^4\Big|_{\frac{\sqrt{3}}{2}}^1 = \frac{1}{8}\left[-\frac{9}{16}+1\right] = \frac{1}{8}\left[+\frac{7}{16}\right] = +\frac{7}{128}$

⑨ $\int_0^1 \frac{x^3\,dx}{(x^4+1)^2}$ let $u = x^4+1$ $x=0 \Rightarrow u=1$

$du = 4x^3$ $x=1 \Rightarrow u=2$

$\frac{1}{4}\int_1^2 \frac{du}{u^2} = -\frac{1}{4u}\Big|_1^2 = -\frac{1}{8}+\frac{1}{4} = \frac{1}{8}$

⑩ $\int_0^{13} \frac{dx}{\sqrt[3]{1+2x}}$ let $u = 1+2x$ $\quad x=0 \Rightarrow u = 1$

$du = 2\,dx \quad x = 13 \Rightarrow u = 27$

$$\frac{1}{2}\int_1^{27} \frac{du}{u^{4/3}} = \frac{1}{2}\cdot\frac{3}{2}u^{\frac{2}{3}}\Big|_1^{27} = \frac{3}{4}(9-1) = \frac{3}{4}\cdot 8 = 6$$

⑪ $\int_0^1 \frac{x^2dx}{1+x^6}$ let $u = x^3$ $\quad \cancel{x} = 0 \Rightarrow u = 0$

$du = 3x^2 \quad x = 1 \Rightarrow u = 1$

$$\frac{1}{3}\int_0^1 \frac{du}{1+u^2} = \frac{1}{3}\tan^{-1}u\Big|_0^1 = \frac{1}{3}\left(\frac{\pi}{4}-0\right) = \frac{\pi}{12}$$

Skip ⑫ $\int_1^2 \frac{e^{\frac{1}{x}}dx}{x^2}$

CRC # 523 $\int \frac{e^{ax}}{x^m}dx = -\frac{1}{m-1}\frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1}\int\frac{e^{ax}}{x^{m-1}}dx$

$$= -\frac{1}{m-1}\frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1}\Big[\ln x + \frac{ax}{1!}$$

⑬ $\int_1^e \frac{\ln^3x}{x}dx$ let $u = \ln x$ $\quad x = 1 \Rightarrow u = 0$

$du = \frac{1}{x}dx \quad x = e \Rightarrow u = 1$

$$\int_0^1 u^3du = \frac{1}{4}u^4\Big|_0^1 = \frac{1}{4}$$

⑭ $\int_0^1 \frac{dx}{(x+1)^3}$ let $\cancel{x} = x+1$ $\quad x = 0 \Rightarrow u = 1$

$du = dx \quad x \Rightarrow 1 \Rightarrow u = 2$

u^{-3} $$\int_1^2 \frac{du}{u^3} = -\frac{1}{2}u^{-2}\Big|_1^2 = -\frac{1}{8}+\frac{1}{2} = \frac{3}{8}$$

Skip, ⑮ $\int_0^2 \sqrt[3]{8-x^2}\,x\,dx$ let $u = 8-x^2$ $\quad x = 0 \Rightarrow u = 8$

$du = -2x\,dx \quad x = 2 \Rightarrow u = 4$

$$-\left(-\frac{1}{2}\right)\int_4^8 u^{\frac{1}{3}}du = \frac{1}{2}\frac{3}{4}u^{\frac{4}{3}}\Big|_4^8 = 6 - \frac{3\sqrt[3]{4}}{2}$$

326

SKIP

⑯ $$\lim_{x\to 0}\left[\frac{1}{\ln(1+x)}-\frac{1}{\sin x}\right]=\lim_{x\to 0}\left[\frac{\sin x-\ln(1+x)}{\underset{u}{\sin x}\,\underset{v}{\ln(1+x)}}\right]\quad \frac{0}{0}$$

$$\lim_{x\to 0}\left[\frac{\cos x-\frac{1}{1+x}}{\underset{u}{\cos x}\,\underset{v}{\ln(1+x)}+\frac{\sin x}{\ln(1+x)}\begin{matrix}u\\v\end{matrix}}\right]=$$ STILL $\frac{0}{0}$

$$\lim_{x\to 0}\left[\frac{-\sin x+\frac{1}{(1+x)^2}}{-\sin x\ln(1+x)+\frac{\cos x}{1+x}+\frac{\cos x\ln(1+x)-\frac{\sin x}{1+x}}{\ln^2(1+x)}}\right.$$

SKIP

⑰ $$\int_{-1}^{1}x e^{-x^4}\,dx$$

$$\overset{u}{x}\,\overset{v}{e^{-x^4}}\qquad \overset{u}{\tfrac{1}{2}x^2}\,\overset{v}{e^{-x^4}}+\frac{4x^5}{5}e^{-4x}$$

$$e^{-x^4}+4x^4e^{-x^4}\qquad xe^{-x^4}+4x^5e^{-x^4}$$

⑱ obvious

⑲ WTS $$\lim_{a\to -1}\int_1^t x^a\,dx=\int_1^t\frac{1}{x}\,dx$$

if $a\neq -1$, $$\lim_{a\to -1}\left.\frac{x^{a+1}}{a+1}\right|_1^t=\lim_{a\to -1}\left[\frac{t^{a+1}}{a+1}-\frac{1}{a+1}\right]=\lim_{a\to -1}\frac{t^{a+1}-1}{a+1}\qquad\frac{0}{0}$$

Take dn WRT a $$\lim_{a\to -1}$$

(23) $\int_0^{\pi} \cos^2\theta \, d\theta = \int_0^{\pi} 1-\sin^2\theta \, d\theta$

$\pm\sqrt{1-\sin^2} = \cos$

Let $u = \sin\theta$ $\quad$ $u=0 \Rightarrow \sin\theta = 0$

$du = \cos\theta \, d\theta$ $\quad$ $u = \pi \Rightarrow \sin\theta = 0$

JACK $\int_0^{\pi} \cos^2\theta \, d\theta = \int_0^{\pi} \cos\theta \, \underbrace{\cos\theta \, d\theta} = \int_0^{0} \sqrt{1-u^2} \, du$

$u = \sin\theta \Rightarrow u^2 = \sin^2\theta \Rightarrow \pm\sqrt{1-u^2}$ the ± causes the problem.

Jill $\int_0^{\pi} \cos^2\theta \, d\theta = \int_0^{\pi} \frac{1+\cos 2\theta}{2} \, d\theta = \frac{\theta + \sin 2\theta}{2}\Big|_0^{\pi} = \frac{\pi}{2}$

$\theta = 0 \Rightarrow u = 0$ It appears that we must reconsider the interval.

$\theta = \frac{\pi}{2} \Rightarrow u = 1$ Cos is >0 $\quad 0 < \theta < \frac{\pi}{2}$ and $\cos < 0 \quad \frac{\pi}{2} < \theta < \pi$

$\theta = \pi \Rightarrow u = 0$ So Jack's problem must be changed:

$$\int_0^{\pi} \sqrt{1-u^2} \, du + \int_1^{0} -\sqrt{1-u^2} \, du$$

Formula Pg 948 #32

$$u\sqrt{1-u^2} + \sin^{-1}x \Big|_0^1 - \left[u\sqrt{1-u^2} - \sin^{-1}x \Big|_1^0 \right]$$

$$1 + \frac{\pi}{2} - 0 + 0 + \frac{\pi}{2} - 1 = \pi \qquad \left[0 - 0 - \{ \frac{\pi}{2} - 1 \right]$$

∴ Jill was right.

JACK ~~JILL~~ DID this.

pg 326

(24) $\int_{-2}^{1} 2x^2\,dx \overset{?}{=} \int_{-2}^{1} x2x\,dx = \int_{-2}^{1} \sqrt{x^2}\,2x\,dx$

JACK: let $u = x^2$ $\quad x = -2 \Rightarrow u = 4$

$du = 2x\,dx$ $\quad x = 1 \Rightarrow u = 1$

$-\int_{4}^{1} \sqrt{u}\,du$ $\qquad \sqrt{u} = |x| = \pm x$

$$\int_{-2}^{0} -x\,2x\,dx + \int_{0}^{1} x\,2x\,dx$$

Jill was right again.

$\frac{2}{3}x^3\Big|_{-2}^{1}$

$\frac{2}{3} + \frac{16}{3}$

6

Sharp Girl — that Jill.

Parts Pg 1

Pg 326-331

$(uv)' =$

$(uv)' = \underline{v\,du} + u\,dv$

$u\,dv = (uv)' - v\,du$

$\int u\,dv = \underbrace{\int (uv)'}_{2^{nd}\ FTC} - \int v\,du$

$\int u\,dv = uv - \int v\,du$ ✱ "Parts"

Go To Eg 3 on Next Pg.

eg1 $\int \underset{u}{x}\ \underset{dv}{e^{-x}\,dx}$

$u = x \quad dv = e^{-x}dx$

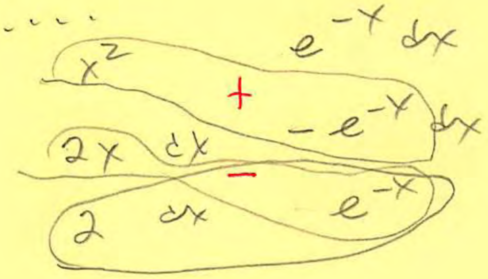
$du = dx \quad v = -e^{-x}dx$

$= -xe^{-x} + \int e^{-x}dx$

$= -xe^{-x} - e^{-x} + c$

short cut.

so....



eg2 $\int \underset{u}{x^2}\, e^{-x}\,dx$

$u = x^2 \quad dv = e^{-x}dx$

$du = 2x\,dx \quad v = -e^{-x}dx$

$-x^2e^{-x} - \int \underset{u}{2x}\ \underset{dv}{e^{-x}dx}$

$u = 2x \quad dv = -e^{-x}$

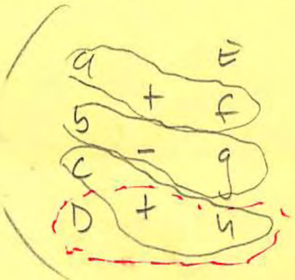
$du = 2\,dx \quad v = e^{-x}$

$-x^2e^{-x} - \left[2xe^{-x} - 2\int e^{-x}dx\right]$

$-x^2e^{-x} - 2xe^{-x} + 2(-e^{-x})$

$-x^2e^{-x} - 2xe^{-x} - 2e^{-x}$

$x^2e^{-x} - 2xe^{-x} - 2e^{-x}$



$Af - bg + ch + Dh = \int a\,E\,dx$

eg 3 $\int \tan^{-1} x \, dx$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{1}{1+x^2}dx \qquad v = x$

$u \quad v \; - \int v\, du$

$x \tan^{-1} x - \int \frac{x\,dx}{1+x^2} \rightarrow$ rather Than actually use Substitution what do you want? a 2? ok

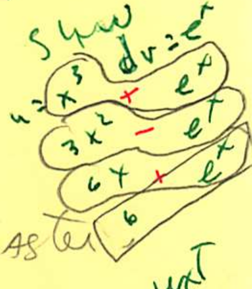
$x \tan^{-1} x - \frac{1}{2}\int \frac{2x\,dx}{1+x^2}$ Just put it in!

$x \tan^{-1} x - \frac{1}{2}\ln(1+x^2)$

NExT do $\int x^3 e^x\,dx$ via parts detailed on NExT page.

$x^3e^x - 3x^2e^x + 6xe^x - 6e^x$

Then show



eg 4 $\int \sin^N x \, dx$

$u = \sin^N x \qquad dv = dx$

$du \; N\sin^{N-1} x \cos x \qquad v = x$

$x \sin^N x - N\int x \sin^{N-1} x \cos x \, dx$

Now what? disaster

I now have an extra x to make Things worse.

NExT eg 4 ←

restart $\int \underline{\sin^N x\,dx} = \int \sin^{N-1} x \cdot \sin x \, dx$

$u = \sin^{N-1} x \qquad dv = \sin x\,dx$

$du = (N-1)\sin x \cos x\,du \qquad v = -\cos x$

$= -\sin^{N-1} x \cos x + (N-1)\int \sin^{N-2} x\, \underline{\cos^2 x}\, dx$

$= + (N-1)\int \sin^{N-2} x\, \underline{(1-\sin^2 x)}\, dx$

$+ (N-1)\int \sin^{N-2} x \, dx - (N-1)\int \underline{\sin^N x \, dx}$

$(N-1)\int \sin^N x \, dx + \int \sin^N x\,dx = -\sin^{N-1} x \cos x + (N-1)\int \sin^{N-2} x\,dx$

$N\int \sin^N x \, dx =$

$\int \sin^N x \, dx = -\frac{1}{N}\sin^{N-1} x \cos x + \frac{(N-1)}{N}\int \sin^{N-2} x \, dx$

$\int x^3 e^x dx$

$u = x^3$ $\quad dv = e^x dx$

$+$

$du = 3x^2 dx$ $\quad v = e^x$

$-$

$du = 6x\, dx$ $\quad v = e^x$

$+$

$du = 6\, dx$ $\quad v = e^x$

$-$

0 $\quad = e^x$

$x^3 e^x - \int 3x^2 e^x dx$

$x^3 e^x - 3x^2 e^x + \int 6x e^x dx$

$x^3 e^x - 3x^2 e^x + 6e^x - \int 6e^x dx$

$-6e^x + C$

using Parts;

$\int x^3 e^x dx$ $\quad u = x^3$ $\quad dv = e^x dx$

$du = 3x^2 dx$ $\quad v = e^x$

$x^3 e^x - \int 3x^2 e^x dx$ $\quad u = 3x^2$ $\quad dv = e^x dx$

$du = 6x\, dx$ $\quad v = e^x$

$x^3 e^x - \left[3x^2 e^x - \int 6x e^x dx \right]$

$x^3 e^x - 3x^2 e^x + \int 6x e^x$ $\quad u = 6x$ $\quad dv = e^x dx$

$du = 6\, dx$ $\quad v = e^x$

$x^3 e^x - 3x^2 e^x + 6x e^x - \int 6e^x dx$

$x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C.$

TAKE [illegible]iv. $\overset{u}{x}\,\overset{\sim}{\tan^{-1}}x - \frac{1}{2}\ln(1+x^2)$

$$\tan^{-1}x + \frac{x}{1+x^2} - \frac{1}{2}\cdot\frac{1}{1+x^2}\cdot 2x = \tan^{-1}x.$$

Thus $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$

$$(uv)' = v\,du + u\,dv$$

so $u\,dv = (uv)' - v\,du$

$$\int \qquad \int \qquad \int$$

$$\int u\,dv = \int (uv)' - \int v\,du$$

$$\Big\downarrow{\scriptstyle FTC}$$

$$\int u\,dv = uv - \int v\,du$$

consider $\int \tan^{-1}x\,dx$ let $u = \tan^{-1}x$ $dv = dx$

$du = \frac{1}{1+x^2}dx$ $v = x$

u

so $\int u\,dv = x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx$

$x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C.$

SYNTh Div

SYNTh Subst.

Factor Thm

~~RAT Root Thm~~

TAke Home Quiz.

$$\int x^3 e^x dx$$

$$\int \frac{1}{x} dx$$

Jim Mumaugh - Vaca High

From: John_Graham@bbns.org
Sent: Wednesday, February 16, 2000 10:25 AM
To: JMumaugh@vusd.solanocoe.k12.ca.us
Subject: Re: RE: Constant of Integration

I think the phrase usually reserved to L'hopital's rule applies. "use it when you should and don't when you shouldn't"

JMumaugh@vusd.solanocoe.k12.ca.us writes:

> This is where the constant should go after the application of "integration by parts". Is that correct?
>
> S 1/x dx = 1 + S 1/x dx + C
>
> If that is correct then my misconception was that I didn't realize that there actually was an integration performed at this stage.

Jim,

Actually, it's been a long time (if ever) since I've analyzed the situation as closely as you obviously have, but now that you've called my attention to it, I realize that every time you do an integration by parts, there has to be an added constant on one side or the other, because the integration-by-parts principle, as I'm sure you know, comes from the product rule for derivatives. Therefore, when you "integrate both sides" of the product rule (and move one term to the other side), a constant of integration is unavoidable.

Thanks for making this point.

John Graham

EXACTLY

this integral generates a constant C_1

call it C_2

thus

$$\int u\,dv + C_1 = uv - \int v\,du + C_2$$

$$C_1 = 1 \pm C_2$$

$$(uv)' = \underline{v\,du} + \underline{u\,dv}$$

$$u\,dv = (uv)' - v\,du$$

$$\int u\,dv = \int (uv)' - \int v\,du$$

FTC

$$\int u\,dv = (uv) - \int v\,du$$

This is where The "constant of integration" appears. no

$dv = dx$

$u = \frac{1}{x}$ $v = x + c$

$du = -\frac{1}{x^2}dx$

etc.

actually the constant is included thusly

Jim Mumaugh - Vaca High

From: John_Graham@bbns.org
Sent: Tuesday, February 15, 2000 6:28 AM
To: JMumaugh@vusd.solanocoe.k12.ca.us
Subject: Re: Constant of Integration

JMumaugh@vusd.solanocoe.k12.ca.us writes:

> I want to be prepared for the time this surfaces in my class and I am not sure where the "constant of integration" is ignored in your demonstration. It seems to me that we "have avoided (or postponed) the integration" by using parts to get the line
>
> S 1/x dx = 1 + S 1/x dx

First of all, I don't think you need to worry about this question ever coming up in your class unless you bring it up. Integration by parts is certainly not the way to integrate 1/x, and I don't think it would naturally occur to a student to do this problem this way.

As a little change of pace, I sometimes ask the class to tell me what's wrong with this calculation, in which I deliberately omit the constant of integration. Someone will say (if I have taught them well and they are alert), "You forgot the constant of integration." (The same omission explains why you can get two apparently different antiderivatives for something like sin x cos x, depending on whether you substitute for sine or cosine.)

After the above comment, which seems to resolve the paradox, I will say, "Well, let's make a definite integral out of it," which of course needs no constant of integration. This brings on a further apparent paradox, because it's often hard for them to see that the constant function 1, evaluated between any a and any b, is 0. This is the ultimate resolution of the paradox, I would say.

Hope this is helpful.

John Graham

Subject: RE: Constant of integration
Date: 01/28 6:37 AM
Received: 01/31 3:22 PM
From: John Graham, John_Graham@bbns.org
Reply-To: Post to AP listserv, ap-calc@ets.org
To: Post to AP listserv, ap-calc@ets.org

ap-calc@ets.org writes:

>You said "for one thing they are sometimes wrong". Are there other
>reasons
>that you are thinking of to eliminate constants of integration?

Speaking of constants of integration, the following puzzle, which I often pose to my class, might be amusing.

$\int \frac{1}{x} dx$

We can integrate the indefinite integral Int(1/x)dx by parts, using u = 1/x, dv = dx. This gives, after a little simplification,

Int(1/x)dx = 1 + Int(1/x)dx

We now subtract the original integral from both sides (a well-known integration-by-parts technique), and obtain

0 = 1.

Students usually see (eventually) that it's because of the suppressed constant of integration that this contradiction is possible, but it's not always immediately clear how changing the problem to a *definite* integral remedies the situation. It does, however.

John A. Graham
Buckingham Browne & Nichols School
Cambridge, MA 02138

$$\int \frac{1}{x}\,dx$$

$u = \frac{1}{x}$ $dv = dx$

$du = \frac{-1}{x^2}\,dx$ $v = x + c$

$uv - \int v\,du$

$$\int \frac{1}{x}\,dx = \frac{1}{x}(x+c) - \int x\left(\frac{-1}{x^2}\right)dx$$

$$\int \frac{1}{x}\,dx = 1 + \frac{c}{x} + \int \frac{1}{x}\,dx$$

$0 = 1$

If $c = 0$ then the seeming paradox is set!

Subject: Increments & Sigma Notation
Date: 01/21 7:59 AM
Received: 01/23 10:23 AM
From: PHILIP GEORGIA, GEORGIA@westosha.k12.wi.us
Reply-To: Post to AP listserv, ap-calc@ets.org
To: Post to AP listserv, ap-calc@ets.org

Hi all,

I am currently teaching approximate area under a curve using Reimann Sums. We have done a number of basic questions like: "Find the area under the curve y=x^2+1 with 3 subintervals over the interval [0,3]."

I now want to lead the kids into an understanding that the integral is the limit of Reimann sums by having them calculate Reimann Sums over increasingly small intervals.

The example I will be giving is: "Find the area under the curve y=x^2+1 using a) 25 subintervals and b) using 100 subintervals over [0,3]." Obviously with so many calculations I am planning on bringing in the calculator at this point.

My left-point answer to the 25 subintervals is to input the following on my TI-83.
(3/25) sum(seq(y1, x, 0, (72/75),(3/25))). Where (3/25) is the width of each subinterval, y1 is the equation x^2 + 1, 0 is the starting point, 72/75 is the ending point and 3/25 is the increment to use when calculating.

My question is, how do I write this in sigma notation on the board (especially the increment of 3/25)? It has been my understanding that when sigma notation is used the increment is always 1.

Right now I am planning to have the kids set up the sum with the normal notation and then just making a note that the increment should be 3/25.

Let me know what you're thinking on this because when I first starting planning these, I was converting the equation so that it would be calculated with an interval of one. (A rather arduous process, (that I can explain further if requested), but good review of sequences and series.)

Hopefully this all makes sense:)

Subject: Re: A fun puzzle
Date: 03/20 4:34 AM
Received: 03/20 7:02 AM
From: AtlntaMath, AtlntaMath@aol.com
Reply-To: ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

Extra Credit
Question
Chang
[illegible]

In a message dated 3/14/1998 11:18:54 AM, you wrote:

<<Here's another good one that I haven't seen here yet:

Consider the function f(x)=1/(x*lnx), where x>1 (for simplicity) and let's look at its anti-derivative using integration by parts.

$\int \frac{1}{x\ln x} dx$

Take u=1/lnx
and dv=(1/x)dx.

$u = \frac{1}{\ln x}$ $dv = \frac{1}{x} dx$

Then du=-1/(lnx)^2 * (1/x)dx
and v=lnx.

$du - \frac{1}{x(\ln x)^2} dx$ $v = \ln x$

Therefore, (S is the integral sign)
S 1/(x*lnx) dx = uv - S v*du = 1/lnx * lnx - S [lnx]*[-1/(lnx)^2 * (1/x)dx]
= 1 + S 1/(x*lnx) dx.

$= 1 + \int \frac{\cancel{\ln x}}{x(\ln x)^2} dx$

Canceling integrals on both sides, we get 0=1.>>

$0 = 1$

Which raises an interesting question that many students seem to miss...If antidifferentiation raises an unknown constant (+C), then at what stage is that constant introduced when performing integration techniques like "Parts" that themselves have integrals that require further simplification? That random constant can be considered applied at any stage of integration. Perhaps the form of "Parts" should instead read:
INT(u dv) + C1 = u*v - INT(v du) + C2
where C1 and C2 are random constants
The above problem then reduces to
C1 = 1 + C2
and the mystery is removed. The problem is not 0=1 as was claimed, but rather that by using integration by parts, you know that the random constant for the right hand side of the equation is one less than the random constant on the left. This is an excellent problem in raising the issue of dealing with constants.

This is the Key!

--Chris Harrow
Mathematics Department
The Westminster Schools
1424 W Paces Ferry Road NW
Atlanta, GA 30327
atlntamath@aol.com

integration by parts.

Pg 326-
331,
Lecture

thm if u & v are differentiable fctns then

$$\int u\,dv = uv - \int v\,du$$

eg 1 $\int \underbrace{x}_{u}\,\underbrace{e^{-x}\,dx}_{dv}$

You must be able to rewrite in the form of $\int u\,dv$

Let $u = x$ $dv = e^{-x}\,dx$
$du = dx$ $v = -e^{-x}$ } this is required!

$$\int u\,dv = -xe^{-x} - \int -e^{-x}\,dx$$

$$-xe^{-x} - e^{-x}$$

$$-e^{-x}[x+1],$$

eg 2 $\int \underset{u}{x^2}\,\underset{dv}{e^{-x}\,dx}$ Let $u = x^2$ $dv = e^{-x}$
$du = 2x\,dx$ $v = -e^{-x}\,dx$

$-x^2e^{-x} + \int \underset{u}{2x}\,\underset{v}{e^{-x}}\,dx$ $2x = u$ $dv = e^{-x}$
$2dx = du$ $v = -e^{-x}$

$-x^2e^{-x} + \left[-2xe^{-x} - \int -2e^{-x}\,dx\right] =$

$-x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}\,dx =$

$-x^2e^{-x} - 2xe^{-x} - 2e^{-x} = -e^{-x}(x^2+2x+2)$

~~is this the only method? No~~

~~$u = x$ $v =$ or $u = e^{-x}$ etc.,~~
~~$dv = xe^{-x}\,dx$ $dv = x^2dx$~~

~~This requires parts of itself~~
~~own~~

Notice that the formula has

"$-\int v\,du$" you don't want the exponent on v to continue to increase since the idea is to end up with $\int du$ some where.

(Next page)

Pg 330
lecture

Conclusion

$$\int \underset{u}{x^n}\, \underset{dv}{e^{ax} dx} \qquad u = x^n \qquad v = \tfrac{1}{a} e^{ax}$$
$$du = n x^{n-1} dx \qquad dv = e^{ax}$$

using $\int u\,dv = uv - \int v\,du$

$$\int x^n e^{ax} dx = x^n e^{ax} - \int \frac{n x^{n-1} e^{ax}}{a} dx$$

which is on pg 949 # 57

eg 3 $\int \underbrace{\tan^{-1} x}_{u}\ \underset{dv}{dx}$ $\qquad u = \tan^{-1} x \qquad v = x$
$\qquad du = \frac{1}{1+x^2} dx \qquad dv = dx$

in lieu of substituting ....

$$x \tan^{-1} x - \int \frac{x}{1+x^2} dx$$

(using Sub method)
what would You like? a 2?
put it in.

$$x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$x \tan^{-1} x - \frac{1}{2} \ln(\underline{1+x^2})$$ ~~which is $x f(x) - \int x f'(x) dx$~~

Ignore
too specific

(this suggests then general formula for this type of integral

$$\int f(x)\, dx = x f(x) - \int f'(x)\, x\, dx$$

eg 4 obtain reduction formula for $\int \sin^n x\ dx$
$u = \sin^n x \qquad v = x$
$du = n \sin^{n-1} x \cos x \qquad dv = dx \qquad$ so

$$\int \sin^n x\, dx = x \sin^n x - n \int x \sin^{n-1} \cos x\, dx$$

this isn't working because although sin is decreasing in powers cos x is increasing so try again ↓

Pg 331
Lecture.

eg 4 (again) $\int \underline{\sin^N x\, dx} = \int \underline{\sin^{N-1} x \sin x\, dx}$

$u = \sin^{N-1} x$ $\qquad v = -\cos x$

$du = N-1 \sin^{N-2} x \cos x\, dx$ $\qquad dv = \sin x\, dx$

$$\int \sin^N x\, dx = \underline{-\sin^{N-1} x \cos x} + \int (N-1) \sin^{N-2} x \underset{(1-\sin^2 x)}{\cos^2 x}\, dx =$$

cos appears again BUT it is cos² so we can replace it ↑

$$-\sin^{N-1} x \cos x + (N-1)\int \sin^{N-2} x \left(1-\sin^2 x\right) dx$$

$$\underline{\int \sin^N x\, dx} = -\sin^{N-1} x \cos x + (N-1)\left[\int \sin^{N-2} x\, dx - \int \sin^N x\, dx\right]$$

$$= -\sin^{N-1} \cos x + N-1 \int \sin^{N-2} x\, dx - \underset{\hookleftarrow}{\underline{(N-1) \int \sin^N x\, dx}}$$

$$N \int \sin^N x\, dx = -\sin^{N-1} \cos x + (N-1) \int \sin^{N-2} x\, dx$$

$$\int \sin^N x\, dx = -\frac{1}{N} \sin^{N-1} \cos x + \frac{N-1}{N} \int \sin^{N-2} x\, dx,$$

eg 5 $\int e^x \cos x\, dx$ $\qquad$ let $u = e^x$ $\qquad v = \sin x$

$du = e^x dx$ $\qquad dv = \cos x\, dx$

$$\int e^x \cos x\, dx = e^x \sin x + \int -\sin x\, e^x dx$$

let $u = e^x$ $\qquad v = \cos x$

$du = e^x dx$ $\qquad dv = -\sin x\, dx$

$$\int e^x \cos x\, dx = e^x \sin x + e^x \cos x - \int \cos x\, e^x dx$$

$$\int e^x \cos x\, dx = \frac{e^x}{2}(\sin x + \cos x)$$

① $\int x e^{2x} dx$ let $u = x$ $v = \frac{1}{2}e^{2x}$

$du = dx$ $dv = e^{2x}$

$\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$

e^{2x}

$2e^{2x}$

$\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} = e^{2x}\left(\frac{2x-1}{4}\right)$

② $\int x^2 e^{2x}dx$ $u = x^2$ $v = \frac{1}{2}e^{2x}$

$du = 2x\,dx$ $dv = e^{2x}$

$\frac{1}{2}x^2e^{2x} - \int xe^{2x}$

(fm prblm 1)

$\frac{1}{2}x^2e^{2x} - e^{2x}\left(\frac{2x-1}{4}\right)$

$\frac{e^{2x}}{4}\left(2x^2 - 2x + 1\right)$

③ $\int x^3 e^{2x}dx$ $u = x^3$ $v = \frac{1}{2}e^{2x}$

$du = 3x^2dx$ $dv = e^{2x}dx$

$\frac{1}{2}x^3e^{2x} - \frac{3}{2}\int x^2e^{2x}dx$

$u = x^3$ $dv = e^{2x}dx$

$\frac{1}{2}x^3e^{2x} - \frac{3}{2}\left[\frac{e^{2x}}{4}\left(2x^2-2x+1\right)\right]$

$3x^2$ $+$ $\frac{1}{2}e^{2x}$

$-$

$6x$ $\frac{1}{4}e^{2x}$

6 $+$ $\frac{1}{8}e^{2x}$

$-$ $\frac{1}{16}e^{2x}$

$\frac{1}{2}x^3e^{2x} - \frac{3}{8}e^{2x}\left(2x^2-2x+1\right)$

$e^{2x}\left(\frac{1}{2}x^3 - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\right)$

$\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} + \frac{3}{4}e^{2x}$

④ $\int \sin^{-1} x \, dx$ $\quad u = \sin^{-1} x \quad v = x$

$du = \frac{1}{\sqrt{1-x^2}} \quad dv = dx$

$x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx \quad u = 1-x^2$

$du = -2x\,dx$

$+\frac{1}{2} \int \frac{du}{u^{1/2}}$

$+\frac{1}{2} \frac{2}{1} u^{\frac{1}{2}}$

$x \sin^{-1} x + \sqrt{1-x^2}$

⑤ $\int x \sin x \, dx$ $\quad u = x \quad v = -\cos x$

$du = dx \quad dv = \sin x$

$-x\cos x + \int \cos x \, dx$

$-x \cos x + \sin x$

⑥ $\int x^2 \sin x \, dx$ $\quad u = x^2 \quad v = -\cos x$

$du = 2x\,dx \quad dv = \sin x$

$-x^2 \cos x + 2\int x \cos x$

$u = x \quad v = \sin x$

$du = dx \quad dv = \cos x$

$-x^2 \cos x + 2\left[x \sin x - \int \sin x \, dx\right]$

$-x^2 \cos x + 2x \sin x + 2 \cos x$

Pg 331

⑦ $\int \ln(4+x^2)\,dx$ $\qquad u = \ln(4+x^2) \qquad v = x$

$du = \frac{2x}{4+x^2} \qquad dv = dx$

or use formula #10 $\longrightarrow \frac{1}{1+(\frac{x}{2})^2}$

$x\ln(4+x^2) - 2\int \frac{x^2}{4+x^2}\,dx$

$x\ln(4+x^2) - 2\left[\int\left(1 - \frac{4}{4+x^2}\right)\right]$

$x\ln(4+x^2) - 2\int dx - 2\int \frac{dx}{1+(\frac{x}{2})^2}$ $\qquad u = \tan^{-1}\frac{x}{2}$

$x\ln(4+x^2) - 2x - 4\tan^{-1}\frac{x}{2}$ $\qquad du = \frac{1}{2}\,\frac{1}{1+\frac{x^2}{4}}$

$2du =$

good prblm ⑧ $\int x\ln x\,dx$

$u = x \qquad v = \overset{u}{x}\,\overset{v}{\ln x} - x$

$du = dx \qquad dv = \ln x$

$\int x\ln x\,dx = x^2(\ln x - 1) - \int \underline{x\ln x}\,dx + \int x\,dx$ $\qquad \underline{\ln x} + 1 - 1$

$\int x\ln x\,dx = \frac{1}{2}\left[x^2\ln x - \frac{x^2}{2} + \frac{x^2}{2}\right]$

$\frac{x^2}{2}\left[\ln x - \frac{1}{2}\right]$

skip ⑨ $\int x(\ln x)^2\,dx$ $\qquad u = x \qquad v = \overset{u}{\frac{x}{3}}\overset{v}{(\ln x)^3}$

$du = dx \qquad dv = (\ln x)^2$

Pg 331

⑩ $\int x^5 e^{x^2} dx$ $\quad u = x^5 \quad v = \frac{1}{2x} e^{x^2}$ too hard, because of not have x times e^{x^2} so

$du = 5x^4 dx \quad dv = e^{x^2}$

$\int x^4 x e^{x^2} dx \quad u = x^4 \quad v = \frac{1}{2} e^{x^2}$

$du = 4x^3 \quad dv = x e^{x^2} dx$ much better.

$\frac{1}{2} x^4 e^{x^2} - 2\int x^3 e^{x^2} \quad u = x^2 \quad v = \frac{1}{2} e^{x^2}$

$du = \underline{2x} dx \quad dv = x e^{x^2}$

$\frac{1}{2} x^4 e^{x^2} - 2\left[\frac{1}{2} x^2 e^{x^2} - \int x e^{x^2}\right]$

$\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + 2\int \underline{x e^{x^2}} dx \quad u = e^{x^2} \quad du = \underline{2x e^{x^2}}$ Simple substitution

$+ \int du \quad \frac{1}{2} e^{x^2}$

$\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + u$

$\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2}$

⑪ $\int e^x \sin x \, dx \quad u = e^x \quad v = -\cos x$

$du = e^x dx \quad dv = \sin x \, dx$

$= -e^x \cos x + \int e^x \cos x \, dx \quad u = e^x \quad v = \sin x$

$du = e^x dx \quad dv = \cos x \, dx$

$= -e^x \cos x + e^x \sin x - \int e^x \sin x$

$= \frac{e^x}{2}(\sin x - \cos x)$

Pg 331

⑫ $\int e^{2x} \cos 3x \, dx \qquad u = e^{2x} \qquad v = \frac{1}{3} \sin 3x$

$du = 2e^{2x} \qquad dv = \cos 3x \, dx$

$= \frac{e^{2x}}{3} \sin 3x - \frac{2}{3}\int e^{2x} \sin 3x$

$u = e^{2x} \qquad v = -\frac{1}{3} \cos 3x$

$du = 2e^{2x} \qquad dv = \sin 3x$

$= \frac{e^{2x}}{3} \sin 3x - \frac{2}{3}\left[\frac{-e^{2x}}{3} \cos 3x + \frac{2}{3}\int e^{2x} \sin 3x \right]$

$= \frac{e^{2x}}{3} \sin 3x + \frac{3}{9} e^{2x} \cos 3x - \boxed{\frac{4}{9}\int e^{2x} \sin 3x}$

$= \frac{9}{13}\left[\qquad \qquad \qquad \right]$

$= \frac{3e^{2x}}{13} \sin 3x + \frac{2}{13} e^{2x} \cos 3x$

⇒ ⑬ $\int x \sec^2 3x \, dx = \int \begin{matrix} u = x \\ du = dx \end{matrix} \quad \begin{matrix} v = \frac{1}{3} \tan 3x \\ dv = \sec^2 3x \, dx \end{matrix}$ improper set up

Good Pblm

$\frac{1}{3} x \tan 3x - \frac{1}{3}\int \tan 3x \, dx$

$-\frac{1}{3}\int \frac{\sin 3x}{\cos 3x} dx \qquad u = \cos 3x$

$du = 3 \sin 3x \, dx$

$+\frac{1}{9}\int \frac{du}{u}$

$\frac{1}{3} x \tan 3x + \ln|u|$

$\frac{1}{3} x \tan 3x + \frac{1}{9} \ln|\cos 3x|$

Pg 331

$$x+4 \overset{1}{\overline{\big) x}}$$
$$\underline{x+4}$$
$$-4$$

⑭ $\int \ln(4+x)\,dx \qquad u = \ln(4+x) \quad v = x$

$\qquad du = \frac{1}{4+x}dx \quad dv = dx$

$$= x\ln(4+x) - \int \frac{x\,dx}{4+x}$$

$$= x\ln(4+x) - \left[\int dx + 4\int \frac{dx}{x+4}\right]$$

$$= x\ln(4+x) - x - 4\ln(4+x)$$

$$= (x-4)\ln(4+x) - x$$

⑮ $\int e^{ax}\sin bx\,dx \qquad u = e^{ax} \quad v = \frac{-1}{b}\cos bx$

$\qquad du = ae^{ax} \quad dv = \sin bx\,dx$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{a}{b}\int e^{ax}\cos bx$$

$\qquad u = e^{ax} \quad v = \frac{1}{b}\sin bx$

$\qquad du = ae^{ax} \quad dv = \cos bx$

$$= \frac{-e^{ax}}{b}\cos bx + \frac{a}{b}\left[\frac{1}{b}e^{ax}\sin bx - \frac{a}{b}\int e^{ax}\sin bx\right]$$

$$= \frac{-e^{ax}}{b}\cos bx + \frac{a}{b^2}e^{ax}\sin bx - \frac{a}{b^2}\int e^{ax}\sin bx$$

$\frac{b^2}{b^2} + \frac{a}{b^2} \quad =$

$\frac{(b^2+a^2)}{b^2} \qquad \frac{b^2}{b^2-a^2}\left[\qquad\qquad\qquad\right]$

$$= \frac{-b}{b^2+a^2}e^{ax}\cos bx + \frac{a}{b^2+a}e^{ax}\sin bx$$

$$\frac{e^{ax}}{b^2+a^2}\left[a\sin bx - b\cos bx\right]$$

Pr 337

⑯ $\int x^2 \cos x \, dx$ $\quad u = x^2 \quad v = \sin x$

$\quad du = 2x\,dx \quad dv = \cos x \, dx$

$x^2 \sin x - 2\int x \sin x \, dx$ $\quad u = x \quad v = \cos x$

$\quad du = dx \quad dv = -\sin x$

$+2\left[x \cos x - \int \cos x \, dx\right]$

$x^2 \sin x + 2x\cos x - 2\sin x$

$N \geq 2$

$$\int \sin^N x \, dx = \frac{\sin^{N-1} x \cos x}{N} + \frac{N-1}{N} \int \sin^{N-2} x \, dx$$

⑰ → a) $\int \sin^2 x \, dx = \frac{1}{2} \sin x \cos x + \frac{1}{2} \int dx$

$\frac{1}{2} \sin x \cos x + \frac{1}{2} x$

→ b) $\int \sin^4 x \, dx = \frac{1}{4} \sin^3 x \cos x + \frac{3}{4} \int \sin^2 x \, dx$

$= \frac{1}{4} \sin^3 x \cos x + \frac{3}{8} \sin x \cos x + \frac{3}{8} x$

Skip c) $\int \sin^6 x \, dx = \frac{1}{6} \sin^5 x \cos x + \frac{5}{6} \int \sin^4 x \, dx$

$= \frac{1}{6} \sin^5 x \cos x + \frac{5}{24} \sin^3 x \cos x + \frac{15}{24} \sin x \cos x$

$+ \frac{15}{24} x$

→ ⑱ a) $\int \sin^3 x \, dx = \frac{1}{3} \sin^2 x \cos x + \frac{2}{3} \int \sin x \, dx$

$= \frac{1}{3} \sin^2 x \cos x - \frac{2}{3} \cos x.$

Pg 332

㉔ $\int \ln(x+1)\,dx$

A) $u = \ln(x+1)$ $\quad v = x$

$du = \frac{1}{x+1}\,dx$ $\quad dv = dx$

$$x+1\overline{)\,x\,}^{\;1}$$
$$\underline{x+1}$$

$$x\ln(x+1) - \int \frac{x\,dx}{x+1}$$

$$x\ln(x+1) - \int dx + \int \frac{dx}{x+1}$$

$$x\ln(x+1) - x + \ln(x+1)$$

$$(x+1)\ln(x+1) - x$$

B) $u = \ln(x+1)$ $\quad v = x+1$

$du = \frac{1}{x+1}\,dx$ $\quad dv = dx$

$$(x+1)\ln(x+1) - \int dx$$

$$(x+1)\ln(x+1) - x$$

c) No comment. which proves, if you look ahead a bit in your pblm you can make this very simple!

Pg 332

(28) $\int_0^1 v(t)\,dt = 1$ D.

A) Distance traveled is the integral of velocity. (accurate)

B) $\int_0^1 a(t)\,dt = 0$

$$\int_0^1 a(t)\,dt = v(t)\Big|_0^1 = v(1) - v(0) = 0$$

v(1) => 0 since velocity at t=1 is zero,
v(0) => 0 since " " t=0 is zero.

C) $\int_0^1 t(a(t))\,dt = -1$

Let $u = t$ $dv = a(t)\,dt$
$du = dt$ $v = v(t)$

$$t\,v(t)\Big|_0^1 - \int v(t)\,dt$$

$1 \cdot v(1) - \int v(t)\,dt$

$0 - (1) = -1$

$\int_0^1 v(t)\,dt = 1$ since dist traveled is one.

(26) ?

* Define "Prime Polynomials"

all 1^{st} degree Poly are Prime.

all 2^{nd} degree Poly with discriminate < 0
are prime poly.

all others are composite polynomials.

Today we will consider
only Prime Second degree
~~equat~~ expressions

* Lecture Begins on Left Page ←

Begin Lecture pg 332-336

#4 $\int \frac{dx}{4x^2+9}$ Notice No du in numerator.

$$\int \frac{dx}{9\left(\frac{4}{9}x^2+1\right)} = \frac{1}{9}\int \frac{dx}{\left(\frac{2}{3}x\right)^2+1}$$

Now it Looks like Tan^{-1}

reciprocal denominator

$$\frac{1}{9}\left(\quad\right) \text{Tan}^{-1}\left(\frac{2}{3}x\right)$$

$$\frac{1}{\cancel{9}_3}\,\frac{\cancel{3}}{2}\,\text{Tan}^{-1}\left(\frac{2}{3}x\right) = \frac{1}{6}\text{Tan}^{-1}\left(\frac{2}{3}x\right)$$

#5   Again No dx on Top

$$\int \frac{8\,dx}{4x^2+8x+13} = \int \frac{8\,dx}{4(x^2+2x+\quad)+13} =$$

complete the square

$$8\int \frac{dx}{4(x^2+2x+1)+13-4} = 8\int \frac{dx}{4(x+1)^2+9} = 8\int \frac{dx}{9\left(\frac{4}{9}(x+1)^2+1\right)}$$

$$\frac{8}{9}\int \frac{dx}{\left(\frac{2(x+1)}{3}\right)^2+1} \Rightarrow \frac{\overset{4}{\cancel{8}}}{\cancel{9}_3}\left(\frac{\cancel{3}}{\cancel{2}}\right)\text{Tan}^{-1}\left(\frac{2(x+1)}{3}\right)$$

$$\frac{4}{3}\text{Tan}^{-1}\left(\frac{2(x+1)}{3}\right)$$

more ↓

Notice part of dx on top

#6 $$\int \frac{x\,dx}{4x^2+8x+13}$$

Since du = 8x+8
create it

$$\frac{1}{8}\int \frac{8x+8\;dx}{4x^2+8x+13} - \int \frac{dx}{4x^2+8x+13}$$

This "corrects" earlier mess

$$\frac{1}{8}\ln(4x^2+8x+13) - \frac{4}{3}\operatorname{Tan}^{-1}\left(\frac{2(x+1)}{3}\right)$$

notice $4x^2+8x+13 > 0 \ \forall x$ since $64 - 16 \cdot 13 < 0$

END of Lecture.

Let students struggle with eg #7, I'll show partial fractions tomorrow which is a better way.

Section 9.6

Very good discussion in Text, Students should read it (pg 332 – 336) Then Begin as a group as long as time allows. **Note margin comment pg 335. this should be shown to be ≥ 0. ie imaginary #s don't have order.**

1.) $\int \frac{dx}{3x-4} = \frac{1}{3}\int \frac{3\,dx}{3x-4} = \frac{1}{3}\ln|3x-4|$

SKip ② $\int$

③ $\int \frac{5\,dx}{(2x+7)^2}$

let $u = 2x+7$
$du = 2\,dx$
$\frac{5}{2}du = 5\,dx$

$\frac{5}{2}\int \frac{du}{u^2} = -\frac{5}{2}u^{-1} \Rightarrow \frac{-5}{2(2x+7)}$

⑤ $\int \frac{dx}{x^2+9}$

let $x^2 = 9u^2 \Rightarrow \int \frac{3\,du}{9u^2+9} = 3\int \frac{1}{9}\frac{du}{u^2+1} =$
$x = 3u$
$dx = 3\,du$

$= \frac{1}{9}\int \frac{dx}{\frac{x^2}{9}+1} = \frac{1}{9}\int \frac{dx}{(\frac{x}{3})^2+1} = \frac{1}{9}\cdot\frac{3}{1}\tan^{-1}\frac{x}{3} =$

$\frac{1}{3}\tan^{-1}\frac{x}{3}$

⑥ $\int \frac{x\,dx}{x^2+9}$

$u = x^2+9$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{2x\,dx}{x^2+9} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\log u = \frac{1}{2}\log(x^2+9)$

⑦ $\int \frac{2x+3}{x^2+9}dx = \int \frac{2x\,dx}{x^2+9} + \int \frac{3}{x^2+9}dx$

$9\cdot\frac{x^2}{9}+1$

$u = x^2+9$
$du = 2x\,dx$

$\int \frac{du}{u} + \frac{1}{\cancel{9}_3}\int \frac{\cancel{3}\,dx}{(\frac{x}{3})^2+1}$

$9u^2 = x^2$

$\ln|u| + \frac{1}{3}\frac{\cancel{3}}{1}\tan^{-1}\frac{x}{3}$

$\ln(x^2+9) + \tan^{-1}\frac{x}{3}$

Think of when You take derivative, ignore the earlier $\frac{1}{3}$. treat it as any constant

Pg 336

$d(\tan^{-1}\sqrt{5}x)$

$\frac{\sqrt{5}}{5x^2+1}$

⑧ $\int \frac{dx}{5x^2+1}$

let $u^2 = 5x^2$

$u = \sqrt{5}x$

$du = \sqrt{5}\,dx$

$\frac{1}{\sqrt{5}}\int \frac{du}{u^2+1} = \frac{1}{\sqrt{5}}\tan^{-1} u = \frac{1}{\sqrt{5}}\tan^{-1} x\sqrt{5}$

⑪ $\int \frac{x\,dx}{2x^2+3}$

let $u = 2x^2+3$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$\frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln(2x^2+3)$

$\frac{1}{a^2-b^2} = \frac{1}{2b}\left(\frac{1}{a-b} - \frac{1}{a+b}\right)$

⑫ $\int \frac{dx}{x^2-1} =$

$\frac{1}{2}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1}$

$\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|$

⑬ $\int \frac{dx}{x^2-4} = \frac{1}{8}\left[\int \frac{dx}{x-2} - \int \frac{dx}{x+2}\right]$

Book Ans wrong!

$\frac{1}{4}\ln|x-2| - \ln|x+2|$

$\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|$

⑮ $\int \frac{dx}{2x^2-3}$ $= \frac{1}{2\sqrt{3}}\left[\int \frac{dx}{x\sqrt{2}-\sqrt{3}} - \int \frac{dx}{x\sqrt{2}+\sqrt{3}}\right]$

$(x\sqrt{2})^2-(\sqrt{3})^2$

Book ans wrong (agn)

$\frac{1}{2\sqrt{3}}\frac{1}{\sqrt{2}}\left[\ln|x\sqrt{2}-\sqrt{3}| - \ln|x\sqrt{2}+\sqrt{3}|\right.$

$\frac{\sqrt{6}}{12}\ln\left|\frac{x\sqrt{2}-\sqrt{3}}{x\sqrt{2}+\sqrt{3}}\right|$

p, 336

⑯ a) x^2+6x+9

$(x+3)^2-9$

b) x^2+7x

$x^2+7x+\frac{49}{4}-\frac{49}{4}$

$\left(x+\frac{7}{2}\right)^2-\frac{49}{4}$

c) x^2-x

$\left(x-\frac{1}{2}\right)^2-\frac{1}{4}$

⑰ $2x^2+8x$

a) $2(x^2+4x+4)-8$

$2(x+2)^2-8$

b) $5x^2+10x$

$5(x^2+2x+1)-5$

$5(x+1)^2-5$

c) $3x^2-7x$

$3\left(x^2-\frac{7}{3}x+\frac{49}{36}\right)-\frac{49}{12}$

$3\left(x-\frac{7}{6}\right)^2-\frac{49}{12}$

51il ⑱ $\int\frac{dx}{9x^2+18x-5} = \int\frac{dx}{9(x^2+2x+1)-14} = \int\frac{dx}{9(x+1)^2-14}$

let $u=x+1$

$du=dx$

$\int\frac{du}{9u^2-14} = \int\frac{du}{(3u)^2-(\sqrt{14})^2} =$

$\frac{1}{2\sqrt{14}}\left[\int\frac{dx}{3u-\sqrt{14}}-\int\frac{dx}{3u+\sqrt{14}}\right.$

Pg 336

⑲ $\int \frac{dx}{4x^2+4x+1} = \int \frac{dx}{4(x^2+x+\frac{1}{4})} = \int \frac{dx}{4(x+\frac{1}{2})^2 -} =$

$\int \frac{dx}{(2(x+\frac{1}{2}))^2 - (\sqrt{2})^2} = \frac{1}{2\sqrt{2}}\left[\int \frac{dx}{2(x+\frac{1}{2})-\sqrt{2}} - \int \frac{dx}{2(x+\frac{1}{2})+\sqrt{2}}\right]$

$\frac{1}{4}\int \frac{dx}{(x+\frac{1}{2})^2}$ $\qquad \frac{\sqrt{2}}{2}\frac{1}{2\sqrt{2}}\,\frac{1}{2}\left[\ln|2x+1-\sqrt{2}| - \ln|2x+1+\sqrt{2}|\right]$

$-\frac{1}{4}(x+\frac{1}{2})^{-1}$ $\qquad \frac{\sqrt{2}}{8}\ln\left|\frac{2x+1-\sqrt{2}}{2x+1+\sqrt{2}}\right|$

⑳ ?

~~$\int \frac{x\,dx}{4x^2+4x-1} =$ let $u = 4x^2+4x-1$, $du = 8x+4\ dx$ $\quad \frac{1}{8}\int \frac{8x+4-4}{4x^2+4x-1}dx$~~

~~$\frac{1}{8}\left[\int \frac{8x\ dx}{4x^2+4x-1}\right] = \frac{1}{8}\left[\int \frac{8x+4}{4x^2+4x-1}dx - \int \frac{4\,dx}{4x^2+4x-1}\right]$~~

~~$\frac{1}{8}\int \frac{du}{u} - \frac{4}{8}\int \frac{dx}{4x^2+4x-1}$~~

~~$\frac{1}{8}\ln u - \frac{4}{8\cdot 4}\int \frac{dx}{x^2+x-\frac{1}{4}}$~~ $\qquad (x^2+x+\frac{1}{4}) - \frac{1}{4} - \frac{1}{4}$

$\qquad (x+\frac{1}{2})^2 - \frac{1}{2}$

~~$\frac{1}{8}\ln(4x^2+4x-1) - \frac{1}{8}\int \frac{dx}{(x+\frac{1}{2})^2 - \frac{1}{2}} = \frac{1}{8}\ln(4x^2+4x-1) - \frac{1}{8\cdot 2}\int \frac{dx}{\frac{(2x+1)^2}{4} - 1}$~~

~~$\frac{1}{8}\ln(4x^2+4x-1) - \frac{1}{2}(x+\frac{\sqrt{2}}{2})\left[\int \frac{dx}{x+\frac{1}{2}-\frac{\sqrt{2}}{2}} - \int \frac{dx}{x+\frac{1}{2}+\frac{\sqrt{2}}{2}}\right]$~~

~~$\frac{1}{8}\ln(4x^2+4x-1) - \frac{1}{2}(x+\frac{\sqrt{2}}{2})\left[\ln|x+\frac{1}{2}-\frac{\sqrt{2}}{2}| - \ln|x+\frac{1}{2}+\frac{\sqrt{2}}{2}|\right]$~~

~~$\frac{1}{8}\ln(4x^2+4x-1) - \frac{1}{2}(x+\frac{\sqrt{2}}{2})\ln\left|\frac{x+\frac{1}{2}-\frac{\sqrt{2}}{2}}{x+\frac{1}{2}+\frac{\sqrt{2}}{2}}\right|$~~

Pg 336

⑳ $\int \frac{x\,dx}{4x^2+4x-1}$ let $u = 4x^2+4x-1$

$du = 8x+4\,dx$

$\frac{1}{8}\int \frac{8x+4-4}{4x^2+4x-1}\,dx = \frac{1}{8}\int \frac{8x+4}{4x^2+4x-1}\,dx - \frac{4}{8}\int \frac{dx}{4x^2+4x-1}$

$\frac{1}{8}\int \frac{du}{u} - \frac{1}{2}\int \frac{dx}{4x^2+4x-1}$

$b^2-4AC \Rightarrow 16+16>0$ ∴

use formula #35 Pg 948

$$\int \frac{dx}{a^2+bx+c} = \frac{1}{\sqrt{b^2-4AC}} \ln\left|\frac{2ax+b-\sqrt{b^2-4AC}}{2ax+b+\sqrt{b^2-4AC}}\right|$$

$a=4$ $b=4$ $c=-1$

$4x^2+4x-1$

$4(x^2+x+\frac{1}{4})-1-1$

$4(x+\frac{1}{2})^2-2$

$4\frac{(2x+1)^2}{4}-2$

$(2x+1)^2-2$

$(2x+1-\sqrt{2})(2x+1+\sqrt{2})$

$\frac{1}{8}\int \frac{du}{u} - \frac{1}{2}\left[\frac{1}{\sqrt{32}} \ln\left|\frac{8x+4-\sqrt{32}}{8x+4+\sqrt{32}}\right|\right]$

$\frac{1}{8}\ln|4x^2+4x-1| - \frac{1}{2}\frac{1}{4\sqrt{2}} \ln\left|\frac{8x+4-4\sqrt{2}}{8x+4+4\sqrt{2}}\right|$

$- \frac{\sqrt{2}}{8} \ln\left|\frac{2x+1-\sqrt{2}}{2x+1+\sqrt{2}}\right|$

p 336

(23) $\int \frac{x-3\,dx}{2x^2+5}$ $u = 2x^2+5$

$du = 4x\,dx$

$$\frac{1}{4}\int \frac{4x-12}{2x^2+5} = \frac{1}{4}\int \frac{4x\,dx}{2x^2+5} - \frac{1}{4}\int \frac{12\,dx}{2x^2+5}$$

$$\frac{1}{4}\int \frac{du}{u} - 3\int \frac{dx}{2x^2+5}$$

$$\frac{1}{4}\ln|u| - 3\cdot\frac{1}{5}\int \frac{dx}{\left(\sqrt{\frac{2}{5}}x\right)^2+1}$$

$$\frac{1}{4}\ln|2x^2+5| - 3\,\frac{\sqrt{5}}{\sqrt{2}}\tan^{-1}\sqrt{\frac{2}{5}}\,x$$

$$\frac{1}{4}\ln|2x^2+5| - \frac{3\sqrt{10}}{10}\tan^{-1}\frac{\sqrt{10}}{5}x$$

$$\int \frac{dx}{(2x+1)^2-1} = 1\int$$

→ $4x^2+4x$

Partial Fractions.

Pg 342

Partial fractions representation of Rational #s

Verify: $\frac{19}{24} = \frac{0}{2^1} + \frac{1}{2^2} + \frac{-1}{2^3} + \frac{2}{3}$

$= \frac{1}{4} - \frac{1}{8} + \frac{2}{3}$

$= \frac{6 - 3 + 16}{24}$

$= \frac{19}{24}$

Notice 1, $19 < 24$

2, $24 = 2 \cdot 2 \cdot 2 \cdot 3$ ie $2^3 \cdot 3$

The method of partial fractions ~~states~~ That ANY rational fraction can be represented as a sum of fractions whose denominators are primes raised to powers. So

$\frac{19}{24} = \frac{a}{2} + \frac{b}{2^2} + \frac{c}{2^3} + \frac{d}{3}$ Now all That is required is to find a, b, c, d

Since my interest is in rational Functions rather Than rational Numbers, I'll show How to find a, b, etc in them.

First, inorder to get This all in The proper perspective, let's review what we know so we may see where we are heading.

Partial fractions.

Pg 337 → 342

Pg 206 lecture

1st a Quick Review.

I Given a pblm like $\int \frac{dx}{(ax+b)}$

we can integrate easily using a substitution of $u = ax+b$ and it causes no problem

II Given a pblm like $\int \frac{dx}{(ax^2+bx+c)}$ where $b^2-4ac<0$

we complete the square then end up with $\tan^{-1}$ usually.

III Given a pblm like $\int \frac{x\,dx}{ax^2+bx+c}$

we make an adjustment in the numerator so $2ax+b = du$ and end up usually with a ln.

I say "usually" above because if the denom. on II or III are raised to powers other than 1 we will have a combination of solutions (ie similar to I & II or I & III.)
generally we won't have troubles like these. They're not all that hard, they just take a lot of time.

* Every rational fctn can be expressed as a sum of a polynomial and some multiple of the above 3 types of fctns.

This is proved in Advanced Calculus. we shall accept it for now and see what it can do,

Partial fractions

Before we actually begin

1. a PRIME fctn has degree one or two.
2. ALL 1^{st} degree polynomials are Prime
3. a 2^{nd} degree polynomial is prime iff $b^2-4ac<0$

As in the representation of rational numbers, rational fctns can be expressed as the sum of prime fctns.

The main difference is that for the rational fctn representation, there is EXACTLY ONE such representation while for rational numbers there are several.

eg $\frac{1}{6} = \frac{-1}{2} + \frac{2}{3}$ $\qquad \frac{1}{6} = \frac{1}{2} - \frac{1}{3}$

5 Steps to solution:

Step 1. Get degree of ~~N~~ Numerator to LESS than degree of Denominator (Divide bottom into top)

Step 2. Factor denom into prime polynomials

Step 3A If $ax+b$ appears exactly n times in the factorization of the Denominator, form the sum

$$\frac{K_1}{ax+b} + \frac{K_2}{(ax+b)^2} + \frac{K_3}{(ax+b)^3} + \dots + \frac{K_n}{(ax+b)^n}, \quad K_i \text{ constants}$$

Pg 4 of
Partial Fract
Lecture

Step 3B If ax^2+bx+c appears exactly m times form the sum:

$$\frac{T_1x+u_1}{ax^2+bx+c}+\frac{T_2x+u_2}{(ax^2+bx+c)^2}+\dots+\frac{T_mx+u_m}{(ax^2+bx+c)^m}$$

Step 4 Determine constants K_i, T_i, u_i

Step 5 integrate

eg #1

$$\int\frac{2x^2+2x-29}{x^2-x-6}\,dx \xrightarrow{\text{Step \#1}} \begin{array}{r@{}l} & \;2 \\ x^2-x-6 & \overline{)\,2x^2+2x-29} \\ & \,\underline{2x^2-2x-12} \\ & \qquad\; 4x-17 \end{array}$$

$$\int 2+\frac{4x-17}{x^2-x-6}\,dx$$

Since anybody can integrate $\int 2\,dx$ I will ignore it for now, and concentrate on $\frac{4x-17}{x^2-x-6}$

Should we "complete the square" on this? No! $b^2-4ac>0$ ∴ not prime.

Step 2 $x^2-x-6=(x-3)(x+2)$

Step 3 $$\frac{4x-17}{x^2-x-6}=\frac{a}{x-3}+\frac{b}{x+2}$$

Step 4 multiply by x^2-x-6 to get

$$4x-17=(x+2)a+(x-3)b$$

Now for the tricky stuff.

we currently have $4x-17 = (x+2)a + (x-3)b$

what if $x=-2$? $-25 = -5b \therefore b=5$
what if $x=3$? $-5 = 5a \therefore a=-1$

So

$$\frac{4x-17}{x^2-x-6} = \frac{-1}{x-3} + \frac{5}{x+2}$$ Thus

$$\int \frac{2x^2+2x-24}{x^2-x-6}\,dx = \int 2\,dx - \int \frac{dx}{x-3} + \int \frac{5\,dx}{x+2}$$

This is clearly easy to integrate so on to eg #2

eg #2

$$\int \frac{6x^2+2x+1}{x^3-3x+2}\,dx$$

1st factor denom

x^3-3x+2

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ \boxed{1} & 1 & 1 & -2 & 0 \end{array}$$

$(x-1)(x^2+x-2)$

$(x-1)(x-1)(x+2)$

So we get

$$\frac{a}{x-1} + \frac{b}{(x-1)^2} + \frac{c}{x+2}$$

$$6x^2+2x+1 = a(x-1)(x+2) + b(x+2) + c(x-1)^2$$

If $x=1$ $9 = +b(3) \therefore b=3$
If $x=-2$ $21 = 9c \therefore c=\frac{7}{3}$
If $x=0$ $1 = -2a+2b+c$
$1 = -2a+6+\frac{7}{3}$
$3 = -6a+18+7$

$\longrightarrow$ $6a = 22$
$a = \frac{11}{3}$

Now finish yourself.

lecture Pg 342

Pg 6 of 6
Lecture

#3

Final example. $\int \frac{x^4+x^3-3x+5}{(x+1)(x^2+x+1)} = A$

Since denominator is x^3 we must divide 1ST.

$$\begin{array}{r|l} & x \quad -1 \\ \hline x^3+2x^2+2x+1 & x^4+x^3 - \quad - \quad -3x+5 \\ & \underline{x^4 \mp 2x^3 \mp 2x^2 \mp x} \\ & -x^3-2x^2-4x \\ & \underline{-x^3 \overset{+}{\mp} 2x^2 \overset{+}{\mp} 2x \overset{+}{\mp} 1} \\ & -2x+6 \end{array}$$

So far!

$$\begin{array}{r} x^2+x+1 \\ \underline{x^3+x^2+x} \\ x^3+2x^2+2x+1 \end{array}$$

$A = x - 1 + \frac{6-2x}{(x+1)(x^2+x+1)}$ notice x^2+x+1 is prime. ie $\sqrt{1-4}$ so

$\frac{6-2x}{(x+1)(x^2+x+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2+x+1}$ ← This is used So deriv of denom will be similar to numerators

Caution: If you use bx rather than bx+c you will NOT notice ANY problems. Everything will seem to work. The only problem is that you will NOT have the proper fractions! ie fatal error.

$6 - 2x = a(x^2+x+1) + (bx+c)(x+1)$

let $x = -1$

$8 = a(1-1+1) + (-b+c)(0)$

$8 = a$

let $x = 0$

$6 = a + (c) \therefore c = -2$

we need one more equation so let $x = 1$ since its relatively easy to work with,

$4 = 3a + 2b + 2c$ but $a = 8$, $c = -2$ so

$4 = 24 + 2b - 4$

$4 = 20 + 2b$

$-16 = 2b$

$b = -8$

↓

Page 8
of Lecture

$$\int \frac{2\,dx}{x^2+x+1}$$

$$2\int \frac{dx}{x^2+x+\frac{1}{4}+\frac{3}{4}} = 2\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = 2\int \frac{dx}{\frac{1}{4}\left[4\left(x+\frac{1}{2}\right)^2+3\right]} =$$

$$\frac{2}{\frac{1}{4}}\int \frac{dx}{\left[2\left(x+\frac{1}{2}\right)\right]^2+3} = 8\int \frac{dx}{(2x+1)^2+3} = \frac{8}{3}\int \frac{dx}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}$$

let $u = \dfrac{2x+1}{\sqrt{3}}$

$du = \dfrac{2}{\sqrt{3}}\,dx$

$\dfrac{\sqrt{3}}{2}\,du = dx$

$$\frac{\overset{4}{\cancel{8}}}{\underset{\sqrt{3}}{\cancel{3}}}\cdot\frac{\sqrt{3}}{\cancel{2}}\int \frac{du}{u^2+1}$$

$$\frac{4}{\sqrt{3}}\tan^{-1}u$$

$$\frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

finally ....

$$\frac{1}{2}x^2 - x + 8\ln|x+1| - 4\ln(x^2+x+1) - \frac{4}{\sqrt{3}}\tan^{-1}\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)$$

so our partial fractions becomes

$$\frac{8}{x+1} - \frac{8x+2}{x^2+x+1}$$

So the Problem now is:

$$\int \frac{x^4+x^3-3x+5}{(x+1)(x^2+x+1)} = \int x-1 + \frac{8}{x+1} - \frac{8x+2}{x^2+x+1}\,dx$$

$$= \frac{1}{2}x^2 - x + 8\ln|x+1| - \int \frac{8x+2}{x^2+x+1}\,dx$$

$$\int \frac{8x+2}{x^2+x+1}\,dx$$

let $u = x^2+x+1$
$du = 2x+1\,dx$

correction integral

$$4\int \frac{2x+1}{x^2+x+1}\,dx - \int \frac{2}{x^2+x+1}\,dx$$

$$4\ln(x^2+x+1) - 2\int \frac{dx}{(x^2+x+\frac{1}{4})+\frac{3}{4}}$$

Done on Last pg.

Put on boards.

6	1
5	2

4	3

QUESTION MARK DAY ACTIVITIES

	Pblm	Board#	
Pg 342	16, 17	#1	4 people
	18	#1 & #2	
	19	#2	2 people
	20	#3	2 people
	21	#3, #4	3/4 people
	22	#5, 6	4/5.

			People
Pg 342	23	#1.	2/3
	24	#2.	2
	25.	#3	3
	27	#5	3
	28	#4	3
	29.	#6.	2/3

collect assignments 10, 11, 12, 13 Begin of period after "Question Mark"

Have them put on Board in 3 waves. as indicated — Everybody putting up each time.

Pg 336 Asgmt # 11	16, 17	#1 ←	3
	11.	#2 ←	2/3
	8.	#3	4
	7.	#3, 4	
	6	#4	2/3
	5.	#5.	4
	1, 3	#6	

Hw (Begins here)

⑯ $\int \frac{x^2+x+3}{x+1}dx$

$$\begin{array}{r} x \\ x+1\overline{)\,x^2+x+3} \\ \underline{x^2+x} \\ 3 \end{array}$$

$\int x\,dx + \int \frac{3}{x+1}dx$ x^2+x+3 is prime.

$\frac{x^2}{2} + 3\ln|x+1|$

⑰ $\int \frac{x}{x-3}dx =$

$$\begin{array}{r} 1 \\ x-3\overline{)\,x} \\ \underline{x-3} \\ 3 \end{array}$$

$\int dx + \int \frac{3\,dx}{x-3}$

$x + 3\ln|x-3|$

⑱ $\int \frac{2x^2+2x+3}{2x+1}dx =$

$$\begin{array}{r} x+\frac{1}{2} \\ 2x+1\overline{)\,2x^2+2x+3} \\ \underline{2x^2+x} \\ x+3 \\ \underline{x+\frac{1}{2}} \\ \frac{5}{2} \end{array}$$

$\int x\,dx + \frac{1}{2}\int dx + \frac{5}{2}\int \frac{dx}{2x+1}$

$\frac{x^2}{2} + \frac{x}{2} + \frac{5}{4}\ln|2x+1|$

⑲ $\int \frac{3x-3}{x^2-9}dx$ $\frac{3x-3}{x^2-9} = \frac{a}{x-3} + \frac{b}{x+3}$

$3x-3 = (x+3)a + b(x-3)$

$x=3$ $6=6a$ $a=1$

$x=-3$ $-12=-6b$ $b=2$

So $\int \frac{dx}{x-3} + 2\int \frac{dx}{x+3}$

$\ln|x-3| + 2\ln|x+3| = \ln|(x-3)(x+3)^2|$

342

20 $\int \frac{x+4}{(x+1)^2} = \frac{a}{x+1} + \frac{b}{(x+1)^2}$

$x+4 = a(x+1) + b$

$x=-1 \quad 3 = b$

$x=0 \quad 4 = a+b \therefore a=1$

$$\int \frac{dx}{x+1} + 3\int \frac{dx}{(x+1)^2} = \ln|x+1| - \frac{3}{(x+1)}$$

(21) $\int \frac{2x^2+9x+12}{(x+2)(x+3)^2} \qquad \frac{a}{x+2} + \frac{b}{x+3} + \frac{c}{(x+3)^2}$

$2x^2+9x+12 = a(x+3)^2 + b(x+2)(x+3) + c(x+2)$

let $x=-3$ $\quad 2 \quad 9 \quad 12$

$-3 \rfloor \quad 2 \quad 3 \quad \textcircled{3} = -c \therefore c=-3$

$x=-2 \quad -2 \rfloor \quad 2 \quad +5 \quad 2 = a$

$x=0 \quad 12 = 9a + 6b + 2c$

$12 = 18 + 6b - 6$

$2 = 3 + b - 1$

$b = 0$

$$2\int \frac{dx}{x+2} - 3\int \frac{dx}{(x+3)^2} = 2\ln|x+2| + \frac{3}{x+3}$$

Pg 342

give students
FINAL ANSwer

22 $\int \frac{x^3}{x^2+3x+5}$

$$\begin{array}{r} x-3 \\ x^2+3x+5 \,\big|\, \overline{x^3 \qquad\qquad} \\ \underline{x^3+3x^2+5x} \\ -3x^2-5x \\ \underline{-3x^2-9x-15} \\ 4x+15 \end{array}$$

$$\int x-3 \; dx + \int \frac{4x+15}{x^2+3x+5}\,dx$$

let $u = x^2+3x+5$
$du = 2x+3$

$$\left(x^2+\frac{3}{}x+\frac{9}{4}\right)-\frac{9}{4}+\frac{20}{4} \quad \frac{1}{4}\left[2\left(x+\frac{3}{2}\right)\right]^2+11$$

$$\frac{1}{4}\left[(2x+3)^2+11\right]$$

$$2\int \frac{2x+3\;dx}{|x^2+3x+5|} + \int \frac{9\,dx}{\left(\frac{2x+3}{\sqrt{11}}\right)^2+1}$$

$$+\int \frac{9\,dx}{\frac{1}{4}\left(\frac{2x+3}{\sqrt{11}}\right)^2+1}$$

$$+\frac{4}{11}\int \frac{9\,dx}{\left[\frac{2x+3}{\sqrt{11}}\right]^2+1}$$

Now

$$\frac{x^2}{2} - 3x + 2\ln|x^2+3x+5| + \frac{9\cdot \overset{2}{\cancel{4}}}{11}\cdot\frac{\sqrt{11}}{\cancel{2}}\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)$$

$$\frac{x^3}{2} - 3x + 2\ln|x^2+3x+5| + \frac{18\sqrt{11}}{11}\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)$$

pg 342

(25) $\int \frac{dx}{x(x-1)(x+1)}$ $\qquad \frac{A}{x} + \frac{b}{x-1} + \frac{c}{x+1}$

$1 = (x-1)(x+1)A + B(x(x-1)) + C(x(x+1))$

let $x=0$ $\quad 1 = -A \therefore A = -1$

$x = 1 \quad 1 = 2C \therefore C = \frac{1}{2}$

$x = -1 \quad 1 = 2B \therefore B = \frac{1}{2}$

$$-\int \frac{dx}{x} + \frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{dx}{x+1}$$

$$-\ln|x| + \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1|$$

$$\ln\left|\frac{(x^2-1)^{1/2}}{x}\right| = \ln\left|\left(\frac{x^2-1}{x^2}\right)\right|^{\frac{1}{2}} = \frac{1}{2}\ln\left|\frac{x^2-1}{x^2}\right|$$

(26) $\frac{1}{a^2-x^2} = \frac{A}{a-x} + \frac{B}{a+x}$

A) $1 = (a+x)A + (a-x)B$ $\qquad$ let $x=-a$

let $x=a$ $\quad 1 = 2aA$ $\qquad -2aB = 1$

$A = \frac{1}{2a}$ $\qquad B = -\frac{1}{2A}$

$\frac{1}{a^2-x^2} = \frac{1}{2a(a-x)} - \frac{1}{2a(a+x)}$ (which is on pg 335)

B) $\frac{1}{2a}\ln\left|\frac{a-x}{a+x}\right| = \frac{1}{2A}\int\left(\frac{1}{a-x} - \frac{1}{a+x}\right)dx$ $\quad$ eg 7,

p. 342

27 $\displaystyle\int \frac{8-4x\,dx}{(x-1)^2(x^2+1)}$

$$\frac{8-4x}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$8-4x = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$

$x=1$ $\quad 4 = 2B \quad \therefore \boxed{B=2}$

$x=0$ $\quad 8 = -A + B + D \quad \therefore \quad \boxed{6 = D - A}\ *$

$x=2$ $\quad 0 = 5A + 5B + (2C+D) \quad \boxed{-10 = 5A + 2C + D}$

$x=-1$ $\quad 12 = -4A + 2B + (D-C)4$

$6 = -2A + 2 + 2D - 2C$

$3 = -A + 1 + D - C$

$2 = -A + D - C$

$*\ -6 = A - D$

$-4 = -C \quad \therefore \boxed{C=4}$

$-10 = 5A + 8 + D$

$-18 = 5A + D$

$-6 = A - D$

$-24 = 6A \quad \therefore \boxed{A=-4}$

$*\ 6 = D - (-4) \quad \therefore \boxed{D=2}$

$$= \int \frac{-4\,dx}{x-1} + \int \frac{2\,dx}{(x-1)^2} + \int \frac{4x+2}{x^2+1}\,dx$$

$$+2\int \frac{2x\,dx}{x^2+1} + \int \frac{2\,dx}{x^2+1}$$

$$= -4\ln|x-1| - \frac{2}{(x-1)} + 2\ln(x^2+1) + 2\,\text{Tan}^{-1} x$$

㉘ $\int \frac{x^4\,dx}{(x^2-1)(x+2)}$

Pg 342

$$\begin{array}{r}
x-2 \\
x^3+2x^2-x-2 \overline{\big) x^4} \\
x^4+2x^3-x^2-2x \\
\hline
-2x^3+x^2+2x \\
-2x^3-4x^2+2x+4 \\
\hline
5x^2+4
\end{array}$$

$$\int x-2\,dx + \int \frac{5x^2+4\,dx}{(x^2-1)(x+2)}$$

$$\frac{5x^2-4}{(x^2-1)(x+2)} = \frac{a}{x-1} + \frac{B}{x+1} + \frac{c}{x+2}$$

$$5x^2-4 = a(x+1)(x+2) + B(x-1)(x+2) + c(x^2-1)$$

$x=1$ $\quad 1 = 6a \quad a = \frac{1}{6}$

$x=-1$ $\quad 1 = -2B \quad b = \frac{-1}{2}$

$x=-2$ $\quad 16 = 3c \quad c = \frac{16}{3}$

$$\int x-2\,dx + \frac{1}{6}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} + \frac{16}{3}\int \frac{dx}{x+2}$$

$$\frac{x^2}{2} - 2x + \frac{1}{6}\ln|x-1| - \frac{1}{2}\ln|x+1| + \frac{16}{3}\ln|x+2|$$

㉔ $\int \frac{x^2\,dx}{x^2+4x+4}$ $\begin{array}{r|l} & 1 \\ \hline x^2+4x+4 & x^2 \\ & x^2+4x+4 \end{array}$ Pg 342

$$= \int dx - \int \frac{4x+4}{(x+2)^2}$$

$$\frac{4x+4}{(x+2)^2} = \frac{a}{x+2} + \frac{b}{(x+2)^2}$$

$$4x+4 = a(x+2) + b$$

$x=-2$ $\quad -4 = b$

$x=0$ $\quad 4 = 2a+b$

$8 = 2a$

$a = 4$

$$\int dx - \int \frac{4\,dx}{x+2} + \int \frac{4\,dx}{(x+2)^2}$$

$$x - 4\ln|x+2| - \frac{4}{x+2}$$

30 Pg 342

A) $\int \frac{2x^3+x\,dx}{x^4+x^2+1} = \frac{1}{2}\int \frac{4x^3+2x\,dx}{x^4+x^2+1} = \frac{1}{2}\ln(x^4+x^2+1)$

B) $\int \frac{dx}{x^4+x^2+1} =$

$$\frac{1}{x^4+x^2+1} = \frac{ax+b}{x^2+x+1} + \frac{cx+d}{x^2-x+1}$$

$$1 = (ax+b)(x^2-x+1) + (cx+D)(x^2+x+1)$$

$x=0$ $\quad 1 = b+D$ $\quad\quad 1 = b+D$

$x=1$ $\quad 1 = (a+b)+3(c+D)$ $\quad\quad 1 = a+b+3c+3D$

$x=-1$ $\quad 1 = 3(-a+b)+(-c+D)$ $\quad\quad 1 = -3a+3b-c+D$

$x=2$ $\quad 1 = 3(2a+b)+7(2c+D)$ $\quad\quad 1 = 6a+3b+14c+7D$

$1-2\sin^2\theta = \cos 2\theta$

$1-\sin^2\theta - \sin^2\theta =$

$\cos^2\theta - \sin^2\theta =$

$\cos 2\theta =$

$1+2\cos^2\theta = \cos 2\theta$

$1+\cos^2\theta+\cos^2\theta =$

$1+(1-\sin^2\theta)+\cos^2\theta =$

$\cos^2\theta-\sin^2\theta =$

$\cos 2\theta =$

Cos add! $\cos 2\theta = \cos^2\theta - \sin^2\theta$

we want $\sin^2\theta$

$\cos 2\theta = 1-2\sin^2\theta$

$2\sin^2\theta = 1-\cos 2\theta$

$\sin^2\theta = \frac{1}{2}-\frac{1}{2}\cos 2\theta$

we want $\cos^2\theta$

$\cos 2\theta = \cos^2\theta-(1-\cos^2\theta)$

$\cos 2\theta = \cos^2\theta - 1 + \cos^2\theta$

$2\cos^2\theta = 1+\cos 2\theta$

$\cos^2\theta = \frac{1}{2}+\frac{1}{2}\cos 2\theta$

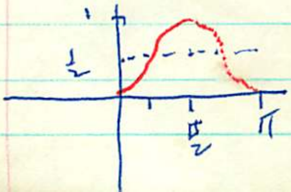
NOTICE

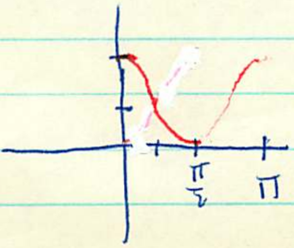
$f(\theta) = -\frac{1}{2}\cos 2\theta + \frac{1}{2}$

Amplitude

$\frac{2\pi}{2} = \pi$ New Period

raise Center line





lecture pg 343 - 350

Prove
~~Verify~~; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \frac{1-[\cos^2\theta - \sin^2\theta]}{2}$

$= \frac{1-\cos^2\theta + \sin^2\theta}{2}$

$= \frac{2\sin^2\theta}{2}$

$= \sin^2\theta$.

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$= \frac{1+\cos^2\theta - \sin^2\theta}{2}$

$= \frac{(1-\sin^2\theta)+\cos^2\theta}{2}$

$= \frac{2\cos^2\theta}{2}$

$= \cos^2\theta$

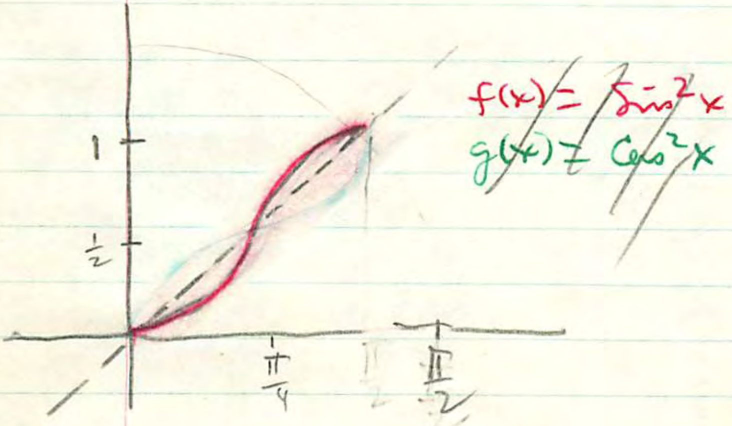
Instead

eg 1 $\int \sin^2\theta \, d\theta$ cannot use substitution method since $u = \sin\theta$ $du = -\cos\theta \, d\theta$, & we don't have a $\cos\theta$ so

$\int \sin^2\theta \, d\theta = \int \frac{d\theta}{2} - \frac{1}{2}\int \cos 2\theta \, d\theta$

$\frac{\theta}{2} - \frac{1}{2}\cdot\frac{1}{2}\sin 2\theta$

$\frac{\theta}{2} - \frac{1}{4}\sin 2\theta$ if you wish the integral not to contain 2θ use sin + formula to clear.

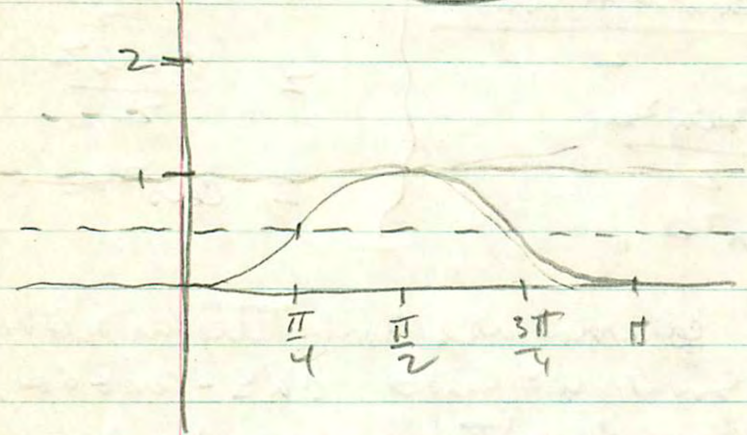


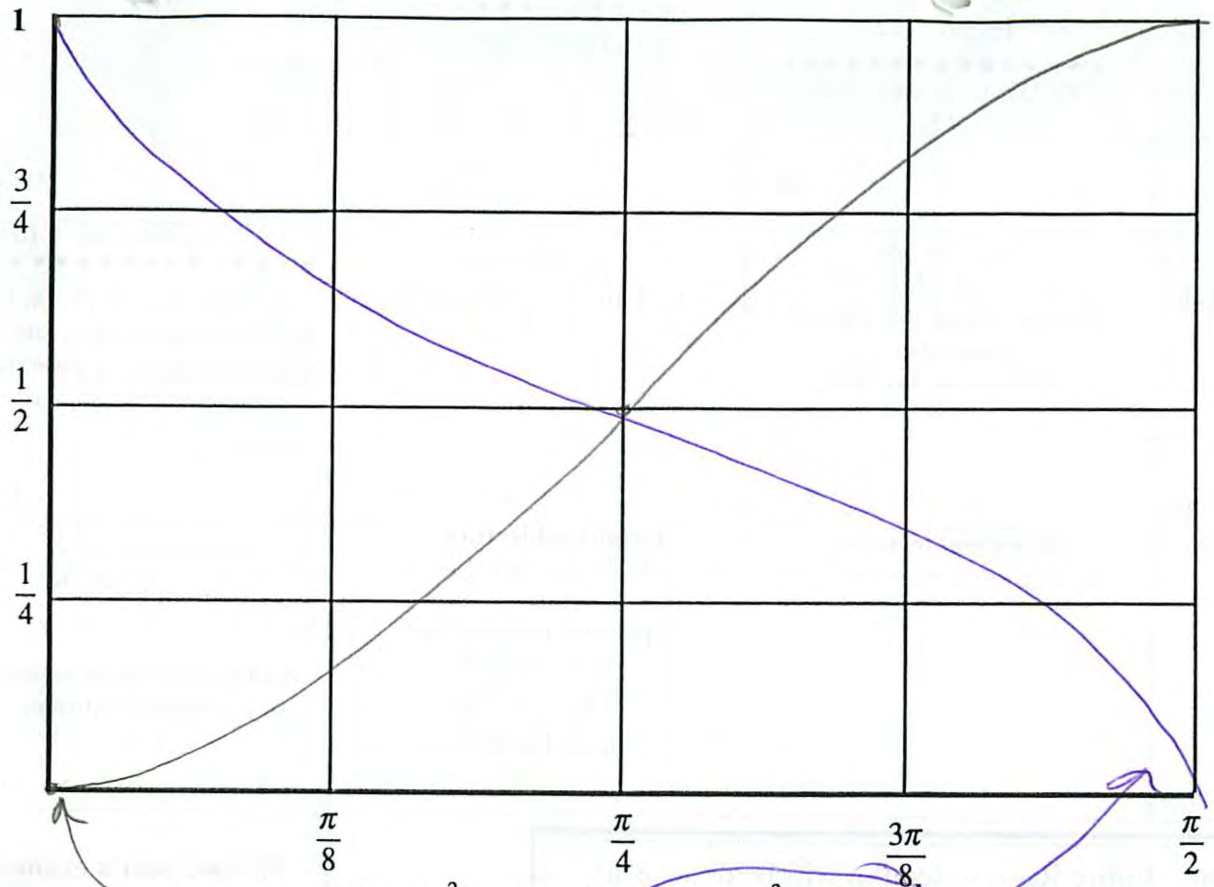
↓

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{1}{2} \underbrace{-\frac{1}{2}}_{\text{amplitude}} \cos 2\theta$$

$\frac{2\pi}{2} = \pi$ = period.





$f(x) = \sin^2 x$ (pencil) $g(x) = \cos^2 x$ (pen)

Rewrite each function in terms of sine or cosine but without any exponents. Give a detailed explanation of how to graph each function *without* the aid of any kind of calculator.

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

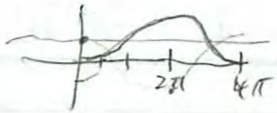
$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$= -\frac{1}{2}\cos 2\theta + \frac{1}{2}$

Amp $= -\frac{1}{2}$

Per $= 4\pi$

raise up $\frac{1}{2}$



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$

$2\cos^2\theta = 1 + \cos 2\theta$

$\cos^2\theta = \frac{1}{2}\cos 2\theta + \frac{1}{2}$

REVIEW SECTION ANSWERS AND/OR HINTS

2) $x \ln x - x$; $\frac{1}{2}(\ln x)^2$

4) let $u = e^x$; $3\sec^2\theta = x^2$

6) $\frac{5}{2}x^2 + 3\ln|x+1| + \ln(x^2+1) - \tan^{-1} x$

12a) $-\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta$

$\tan\theta - \theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta$

18) $x + \frac{1}{x-1} + \frac{x-1}{x^2+x+1}$

20) $1 + \frac{1}{x-1} + \frac{2}{x+1} - \frac{1}{x^2+1}$

25) Briefly explain ALL possibilities

29) Page 351 case 3

30) Page 349 eg 8

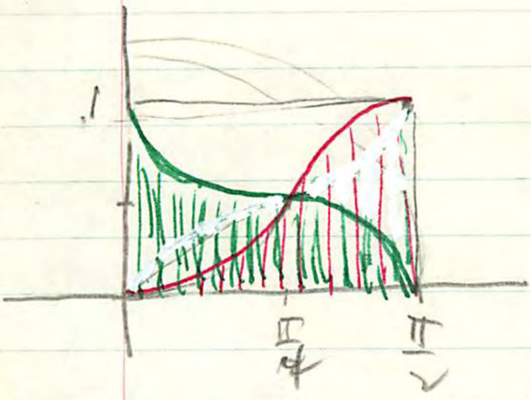
34) $\frac{4}{3}\sqrt{\sqrt{x}+2}\,(\sqrt{x}+4)$

43) $\sin 3x \;\cos 3x = \frac{1}{2}\sin 6x$

46) $\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1} x$

150) So, who needs tables, anyway?

P_r 343-350
Lecture



$g(\theta) = \cos^2\theta$

$f(\theta) = \sin^2\theta$

area of rectangle is $\frac{\pi}{2}$

It appears that f and g have the same area i.e. $\frac{1}{2}$ of the rectangle. indeed they do.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\int_0^{\frac{\pi}{2}} \sin^2\theta\,d\theta + \int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = \int_0^{\frac{\pi}{2}} 1\theta = \frac{\pi}{2}$$

So remember!

$$\int_0^{\frac{\pi}{2}} \sin^2\theta\,d\theta = \frac{\pi}{4} = \int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta$$

Notice exponent is "odd"

eg 2 $\int \sin^5\theta\,d\theta$ Since $d(\cos\theta) = -\sin\theta\,d\theta$ and $\sin^4\theta\,\sin\theta = \sin^5\theta$

$$\int \sin^5\theta\,d\theta = \int \underline{\sin^4\theta}\,\underline{\sin\theta\,d\theta} = \int (1-\cos^2\theta)^2 \underline{\sin\theta\,d\theta}$$

Now let $u = \cos\theta$ so $du = -\sin\theta\,d\theta$ so,

$$-\int (1-u^2)^2\,du = -\int 1 - 2u^2 + u^4\,du =$$

$$-\left(u - \frac{2}{3}u^3 + \frac{u^5}{5}\right) = -\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta.$$

Lectures Pg 343-350

eg 3 $\int \underline{\cos^3\theta} \sin^4\theta \, d\theta$ "odd man out" method.

$\int \cos^2\theta \sin^4\theta \, \underline{\cos\theta \, d\theta}$ Because "extra" in cos we want u to be sin. so we have a du,

$\int (1-\sin^2\theta) \sin^4\theta \cos\theta \, d\theta$

$\int (\sin^4\theta - \sin^6\theta) \cos\theta \, d\theta$ Let $u = \sin\theta$
$du = +\cos\theta \, d\theta$

$\int u^4 \, d\theta - \int u^6 \, d\theta$

$\frac{u^5}{5} - \frac{u^7}{7} \Rightarrow +\frac{1}{5}\sin^5\theta + \frac{1}{7}\sin^7\theta$

eg 4 $\int \tan^6\theta \sec^4\theta \, d\theta$ $d(\tan\theta) = \sec^2\theta \, d\theta$

Both even
$s^2+c^2=1$
$t^2+1=\sec^2$

$\int$ $\underline{\sec^2\theta \, d\theta}$ Now can I rewrite the remaining $\sec^2\theta$ in terms of $\tan\theta$?
Sure $\tan^2\theta = \sec^2\theta - 1$ so

$\int \tan^6\theta \, (\underline{\tan^2+1}) \sec^2\theta \, d\theta$ Let $u = \tan\theta$
$du = \sec^2\theta \, d\theta$

$\int u^8 + u^6 \, du = \frac{1}{9}u^9 + \frac{1}{7}u^7 \Rightarrow \frac{1}{9}\tan^9\theta + \frac{1}{7}\tan^7\theta.$

Conclusion if Power of $\underline{\tan \text{ \& } \sec}$ are even this set up will work easily.

Both odd

when power of $\tan\theta$ is odd we cannot pair up the $\sec^2\theta$ so ;

Pg 343-350

eg 5 $\int \tan^5\theta \sec^3\theta \, d\theta$ $d(\sec\theta) = \sec\theta\tan\theta\, d\theta$

$\int \tan^4\theta \sec^2\theta \sec\theta\tan\theta \, d\theta$ Now to make Tan into Sec.

$\int (\sec^2\theta - 1)^2 \sec^2\theta \, \underline{\sec\theta\tan\theta} \, d\theta$

$\int (u^2-1)^2 u^2 \, du$

$\int u^4 - 2u^3 + u^2 \, du$

$\frac{1}{5}u^5 - \frac{1}{2}u^4 + \frac{1}{3}u^3 \Rightarrow$

$\frac{1}{5}\sec^5\theta - \frac{1}{2}\sec^4\theta + \frac{1}{3}\sec^3\theta$.

eg 6

↓

Skip This page

Pr 343-350 lecture

$$\int \tan^N d\theta = \int \tan^{N-2}\theta(\sec^2\theta - 1)\,d\theta$$

let $u = \tan\theta$

$du = \sec^2\theta$

$$\int u^{N-2}\,du - \int u^{N-2}\,du$$

$$\frac{u^{N-1}}{N-1} - \int u^{N-2}\,du$$

$$- \int u^{N-4}(\sec^2\theta - 1)\,d\theta$$

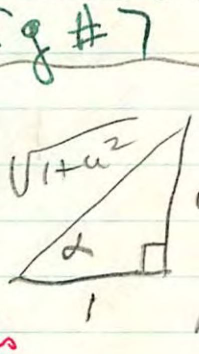
etc,

if N is even eventually we up with $\tan^0\theta$ so $\int \tan^0\theta\,d\theta = \int d\theta = \theta$

if N is odd we will end with $\int \tan\,d\theta$ which can be handled $\int \frac{\sin\theta}{\cos\theta}\,d\theta = -\ln|\cos\theta|$

Skip to Eg #7

Not really necessary until next section.



Now $\cos\alpha = \frac{1}{\sqrt{1+u^2}}$, $\sin\alpha = \frac{u}{\sqrt{1+u^2}}$, $\tan\alpha = u$

(Labels were chosen because of $\tan\alpha = u$)

this method is Truely the "Last resort"! generally it yields a terrible fraction. It will have an elementary integral but it will also be a super mess

Now $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$$= \left(\frac{1}{\sqrt{1+u^2}}\right)^2 - \left(\frac{u}{\sqrt{1+u^2}}\right)^2$$

$$\cos 2\alpha = \frac{1-u^2}{1+u^2}$$

Also $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\frac{u}{\sqrt{1+u^2}} \cdot \frac{1}{\sqrt{1+u^2}} = \frac{2u}{1+u^2}$$

In order to have cos & sin in terms of single angle we make the substitution $2\alpha = \theta$, ie $\alpha = \frac{\theta}{2}$ to get ↓

Skip this page too pg 343-350 lecture,

$\cos\theta = \frac{1-u^2}{1+u^2}$, $\sin\theta = \frac{2u}{1+u^2}$, $\tan\frac{\theta}{2} = u$

Next consider $\tan\frac{\theta}{2} = u$

$$du = \frac{1}{2}\sec^2\theta \, d\theta$$

$$= \frac{1}{2}(1+\tan^2\theta)d\theta$$

$$du = \frac{1}{2}(1+u^2)\, d\theta$$

$$\frac{2\,du}{1+u^2} = d\theta$$

So the complete list is: Remember all this is based on

$\cos\theta = \frac{1-u^2}{1+u^2}$, $\sin\theta = \frac{2u}{1+u^2}$, $\tan\frac{\theta}{2} = u$, $d\theta = \frac{2\,du}{1+u^2}$

Now to use this:

eg 8 $\int \frac{\sin^3 \cos\theta \, d\theta}{1+2\cos\theta}$ ⟹ so let $u = \tan\frac{\theta}{2}$

$$= \int \frac{\left(\frac{2u}{1+u^2}\right)^3 \left(\frac{1-u^2}{1+u^2}\right) \frac{2\,du}{1+u^2}}{1+2\left(\frac{1-u^2}{1+u^2}\right)} = \int \frac{\frac{16u^3(1-u^2)du}{(1+u^2)^5}}{\frac{1+u^2+2-2u^2}{(1+u^2)}} =$$

$$\int \frac{16u^3(1-u^2)(1+u^2)\,du}{(1+u^2)^5(3-u^2)} = \int \frac{16u^3(1-u^2)\,du}{(1+u^2)^4(3-u^2)}$$

this could be solved using partial fractions, we will not do it now because of the length of time required.

end of lecture.

$\int \sec\theta \, d\theta$

$\int \frac{d\theta}{\cos\theta}$

$\int \frac{\cos\theta \, d\theta}{\cos^2\theta}$

$\int \frac{\cos\theta \, d\theta}{1-\sin^2\theta} = \int \frac{du}{1-u^2}$ $\qquad \frac{A}{1-u} + \frac{B}{1+u}$

Let $u = \sin\theta$

$du = \cos\theta \, d\theta$

$1 = A(1+u) + B(1-u)$

If $u=1$ $\quad 1 = 2A \therefore A = \frac{1}{2}$

$u=-1$ $\quad 1 = +2B \quad B = \frac{1}{2}$

Der of inside

$\frac{1}{2}\int \frac{du}{1-u} + \frac{1}{2}\int \frac{du}{1+u}$

$\ominus\frac{1}{2} \ln|1-u| + \frac{1}{2}\ln|1+u|$

$\frac{1}{2} \ln\left|\frac{1+u}{1-u}\right|$

$u = \sin\theta$ $\qquad \frac{1}{2} \ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right| \left|\frac{1+\sin\theta}{1+\sin\theta}\right|$

$\frac{1}{2} \ln \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$

$\frac{1}{2} \ln \frac{(1+\sin\theta)^2}{\cos^2\theta}$

$\ln\left|\frac{1+\sin\theta}{\cos\theta}\right| \to \ln|\sec\theta + \tan\theta|$

eg 7

Pg 343-350
Lecture.

$$\int \sec\theta\, d\theta = \int \frac{d\theta}{\cos\theta} = \int \frac{\cos\theta\, d\theta}{\cos^2\theta} =$$

$$\int \frac{\cos\theta\, d\theta}{1-\sin^2\theta} \qquad \text{let } u = \sin\theta$$
$$du = \cos\theta\, d\theta$$

$$\int \frac{du}{1-u^2} \Rightarrow \frac{a}{1-u} + \frac{b}{1+u}$$

$$1 = (1+u)a + (1-u)b$$

$$u = 1 \qquad 1 = 2a \quad \therefore a = \frac{1}{2}$$

$$u = -1 \qquad 1 = 2b \quad \therefore b = +\frac{1}{2}$$

$$\int \frac{du}{1-u^2} = \frac{1}{2}\int \frac{du}{1-u} + \frac{1}{2}\int \frac{du}{1+u}$$

Don't neglect

$$\boxed{-}\frac{1}{2} \ln|1-u| + \frac{1}{2} \ln|1+u|$$

$$\frac{1}{2} \ln\left|\frac{1+u}{1-u}\right|$$

$$\frac{1}{2} \ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right|$$

abs not needed. $\sin\theta < 1$

But are needed after applying the $\frac{1}{2}$?

why? I know I don't. "Sq root of something squared"?

$$\frac{1}{2} \ln\left(\frac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta}\right)$$

$$\frac{1}{2} \ln \frac{(1+\sin\theta)^2}{\cos^2\theta}$$

$$\ln \frac{1+\sin\theta}{\cos\theta}$$

$$\ln|\sec\theta + \tan\theta|$$

Pg 350

This should have been a +

#1 answer given is

$-\frac{1}{5}\overset{u}{\cos\theta}\,\overset{v}{\sin^4\theta} - \frac{4}{5}\cos\theta - \frac{4}{15}\cos^3\theta$

$\frac{1}{5}\sin^5\theta - \frac{4}{5}\sin^3\theta\cos^2\theta + \underbrace{\frac{4}{5}\sin\theta - \frac{4}{5}\cos^2\theta\sin\theta}$

$\frac{4}{5}\sin\theta\,(1-\cos^2\theta)$

$\frac{4}{5}\sin^3\theta$

$\underbrace{\qquad\qquad}$

$\frac{4}{5}\sin^3\theta\,(-\cos^2\theta+1)$

$\frac{4}{5}\sin^5\theta$

$\underbrace{\qquad\qquad}$

$\sin^5\theta$

$-\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta$

$\sin\theta + 2\cos^2\theta(-\sin\theta) + \cos^4\theta\sin\theta$

Pg 350 #1

$-\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta$

$\sin\theta - 2\sin\theta\cos^2\theta + \sin\theta\cos^4\theta$

$\sin\theta(1 - 2\cos^2\theta + \cos^4\theta)$

$\sin\theta(1 - \cos^2\theta)^2$

$\sin\theta(\sin^2\theta)^2$

$\sin^5\theta$

The Problem:

$\int \sin^5\theta \, d\theta$

$\int \sin^4\theta \sin\theta \, d\theta$

$\int (1-\cos^2\theta)^2 \sin\theta \, d\theta$

let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$-\int (1-u^2)^2 \, du$

$-\int 1 - 2u^2 + u^4 \, du$

$-(u - \frac{2}{3}u^3 + \frac{1}{5}u^5)$

$\rightarrow -(\cos\theta - \frac{2}{5}\cos^3\theta + 1\cos^5\theta)$

$-\cos\theta + \frac{2}{5}\cos^3\theta - \frac{1}{5}\cos^5\theta.$

Pg 350 #1 answer in text has minus, should be +.

$$-\frac{1}{5}\overset{u}{\sin^4\theta}\,\overset{v}{\cos\theta} - \frac{4}{5}\cos\theta + \frac{4}{15}\cos^3\theta$$

$$-\frac{4}{5}\cos^2\theta\sin^3\theta + \frac{1}{5}\sin^5\theta + \underbrace{\frac{4}{5}\sin\theta - \frac{4}{5}\sin\theta\cos^2\theta}$$

$$\underline{-\frac{4}{5}\sin^3\theta\cos^2\theta}$$

$$\frac{4}{5}\sin\theta\,(1-\cos^2\theta)$$

$$\frac{4}{5}\sin\theta\cdot\sin^2\theta$$

$$\underline{\frac{4}{5}\sin^3\theta}$$

$$\frac{4}{5}\sin^3\theta\,(-\cos^2\theta+1)$$

$$\frac{4}{5}\sin^3\theta\,\sin^2\theta$$

$$\underbrace{\frac{4}{5}\sin^5\theta + \frac{1}{5}\sin^5\theta}$$

$$\sin^5\theta.$$

Pg 350

① $\int \sin^5\theta\,d\theta = \int \sin^4\theta \sin\theta\,d\theta =$

$\int (1-\cos^2\theta)^2 \sin\theta\,d\theta$, let $\cos\theta = u$
$du = -\sin\theta\,d\theta$

$-\int (1-u^2)^2\,du = -\int 1-2u^2+u^4\,d\theta =$

$-u + \frac{2}{3}u^3 - \frac{1}{5}u^5 \Rightarrow -\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta$

Book answer ~~wrong~~, no different form

⟸

The true criteria is "If when you take the Der of it, you get the proper items, you have the proper answer"

② $\int \sin^2\theta\cos^3\theta\,d\theta = \int \sin^2\theta\cos^2\theta\cos\theta\,d\theta =$

$\int \sin^2\theta(1-\sin^2\theta)\cos\theta\,d\theta$ let $u = \sin\theta$ $d\theta = \cos\theta\,d\theta$

$\int u^2-u^4\,du = \frac{1}{3}u^3-\frac{1}{5}u^5 \Rightarrow \frac{1}{3}\sin^3\theta - \frac{1}{5}\sin^5\theta$

③ $\int \tan^4\sec^2\theta\,d\theta$ let $u = \tan\theta$
$du = \sec^2\theta\,d\theta$

$\int u^4\,du = \frac{1}{5}u^5 \Rightarrow \frac{1}{5}\tan^5\theta$

④ $\int \tan^5\sec^3\theta\,d\theta = \int \tan^4\theta\sec^2\theta\,\underbrace{\sec\theta\tan\theta\,d\theta}_{\uparrow\ du}$

$s^2+c^2=1$
$T^2+1=\sec^2$
$u=\sec\theta$

$\int (\sec^2\theta-1)^4\sec^2\theta\sec\theta\tan\theta\,d\theta = \int (u^2-1)^2u^2\,du =$

$\int u^6-2u^4+u^2\,du = \frac{1}{7}u^7-\frac{2}{5}u^5+\frac{1}{3}u^3 \Rightarrow$

$\frac{1}{7}\sec^7\theta - \frac{2}{5}\sec^5\theta + \frac{1}{3}\sec^3$

P 350

⑤ $\int \sin^3 \cos^4\theta \, d\theta = \int \sin^2\theta \cos^4\theta \sin\theta \, d\theta$

Let $u = \cos\theta$
$du = -\sin\theta \, d\theta$ $-\int (1-u^2) u^4 du = -\int u^4 - u^6 \, du$

$-\frac{1}{5} \cos^5\theta + \frac{1}{7} \cos^7\theta$.

⑥ $\int \csc\theta \, d\theta = \int \frac{1}{\sin\theta} d\theta = \int \frac{\sin\theta}{\sin^2\theta} d\theta = \int \frac{\sin\theta \, d\theta}{1-\cos^2\theta}$

Let $u = \cos\theta$
$du = -\sin\theta \, d\theta$ $-\int \frac{du}{1-u^2} = \int \frac{du}{u^2-1} =$

$\frac{1}{2} \int \frac{1}{u-1} - \frac{1}{u+1} \, du = \frac{1}{2} \ln \left| \frac{u-1}{u+1} \right|$

$\frac{1}{2} \ln \left| \frac{\cos\theta - 1}{\cos\theta + 1} \right|$

⑦ $\int \cot\theta \, d\theta = \int \frac{\cos\theta \, d\theta}{\sin\theta}$ Let $u = \sin\theta$
$du = \cos\theta \, d\theta$

$\int \frac{du}{u} = \ln|\sin\theta|$

⑧ $\int \cos^2 3\theta \, d\theta = \frac{1}{2} \int 1 + \cos 6\theta \, d\theta =$

$\frac{\theta}{2} + \frac{1}{12} \sin 6\theta$

Integrals involving trig ~~substitutions~~
Pg 343-350
Lecture

$$\int \cos^2\theta = \frac{1}{2}\int \theta + \cos 2\theta \, d\theta$$

$$\int \sin^2\theta = \frac{1}{2}\int \theta - \sin 2\theta \, d\theta$$

$$\int (\sin/\cos)^{odd} \to \int (\text{opposite})^{even} \underbrace{(\sin/\cos)\, d\theta}_{du}$$

$\int \sin^a\theta \cos^b\theta \, d\theta$ if one is odd make it even and use extra for du convert all else to other fct $\cos^3\theta \Rightarrow (1-\sin^2\theta)$

$\int \tan^a\theta \sec^b\theta$ if b is even ∴ use $\sec^2\theta \, d\theta$ as du make all else tan.

if a and b are odd make $\sec\theta \tan\theta \, d\theta$ as du convert to $\sec\theta$ s.

$\int \sec\theta \, d\theta \Rightarrow \ln|\sec\theta + \tan\theta|$

$S+C=1$

$T+1=\sec$

pg 350

⑨ $\int \tan^4 d\theta = \int \tan^2\theta \tan^2\theta \, d\theta = \int \tan^2\theta(\sec^2\theta - 1) \, d\theta$

$\int \tan^2\theta \sec^2\theta - \tan^2\theta \, d\theta$

Let $u = \tan\theta$

$du = \sec^2\theta$

$\int u^2 du - \int \sec^2\theta - 1 \, d\theta$

$\frac{1}{3}u^3 - \tan\theta + \theta \Rightarrow \frac{1}{3}\tan^3\theta - \tan\theta + \theta.$

⑩ $\int \sec 3\theta \, d\theta = \int \frac{1}{\cos 3\theta} d\theta = \int \frac{\cos 3\theta}{\cos^2 3\theta} d\theta =$

$\int \frac{\cos 3\theta \, d\theta}{1-\sin^2 3\theta}$ Let $u = \sin 3\theta$

$du = 3\cos 3\theta.$

$\frac{1}{3}\int \frac{du}{1-u^2} = \frac{1}{6} \ln\left|\frac{1+\sin 3\theta}{1-\sin 3\theta}\right|$

$s^2+c^2=1$

$1+\cot^2=\csc^2$

⑪ $\int \cot^3\theta \csc^5\theta \, d\theta = \int \cot^2\theta \csc^4\theta \cot\theta \csc\theta \, d\theta$

$u = \csc\theta$

$du = -\csc\theta\cot\theta$ $-\int (u^2-1) u^4 du = -\int u^6 - u^4 du$

$-\frac{1}{7}\csc^7\theta + \frac{1}{5}\csc^5\theta.$

350

$s^2+c^2=1$
$1+\cot^2=\csc^2$

⑫ $\int \cot^4\theta \csc^4\theta \, d\theta = \int \cot^4\theta \csc^2\theta \csc^2\theta \, d\theta$

let $u = \cot\theta$
$du = -\csc^2\theta$
$-\int u^4(1+u^2)\,du = -\int u^4+u^6\,du$

$-\frac{1}{5}\cot^5\theta - \frac{1}{7}\cot^7\theta$

⑬ $\int (\sin\theta + 2\cos\theta)^2 \, d\theta = \int \sin^2\theta + 4\sin\theta\cos\theta + 4\cos^2\theta \, d\theta$

$\int \sin^2\theta + 4\cos^2\theta \, d\theta + 4\int \sin\theta\cos\theta \, d\theta$
$u = \sin\theta \quad du = \cos\theta\, d\theta$

$\int 1 + 3\cos^2\theta \, d\theta + 2\sin^2\theta$

$\int 1 + \frac{3}{2}(1+\cos 2\theta)\, d\theta +$

$\theta + 2\sin^2\theta + \frac{3}{2}\int 1 + 2\cos 2\theta \, d\theta$

$+\frac{3}{2}\theta + 3\int \cos 2\theta \, d\theta$

$\frac{5}{2}\theta + 2\sin^2\theta + \frac{3}{2}\sin 2\theta$

⑯ $\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta = \frac{\pi}{4}$

⑲ $\int \frac{d\theta}{1+\cos\theta}$ $\quad \cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$

$\int \frac{d\theta}{2\cos^2\theta} = \frac{1}{2}\int \sec^2\frac{\theta}{2}\, d\theta = 2\cdot\frac{1}{2}\tan\frac{\theta}{2} = \tan\frac{\theta}{2}$

Trig Substitutions

$\sin^2\theta + \cos^2\theta = 1$ leads to:

$\int \sqrt{1-x^2}\,dx$

$\cos^2\theta = 1 - \sin^2\theta$

$\Updownarrow$

$1 - x^2$

$\int \sqrt{x^2+1}\,dx$

$t^2 + 1 = \sec^2\theta$

$\sec^2\theta = \tan^2\theta + 1$

$\Updownarrow$

$x^2 + 1$

$\int \sqrt{x^2-1}\,dx$

$\tan^2\theta = \sec^2\theta - 1$

$\Updownarrow$

$x^2 - 1$

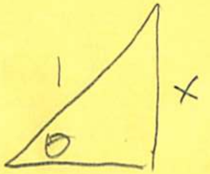
So to remove the √ in an integral.

Let $x = \sin\theta$

to get

$\cos\theta$

So we get ∫cos thus NOT having the Negative sign

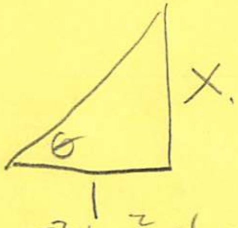


Because d(Sin) = Cos and dx(Cos) = − Sin we can avoid the Negative by selecting to ∫ cos

Let $x = \tan\theta$

to get

$\sec\theta$



$s^2 + c^2 = 1$

This could be

$\tan^2 + 1 =$ or

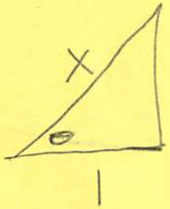
$1 + \cot^2 =$

we select $\tan^2 + 1$ which is $\sec^2$ and we know ∫sec² is ln Sec+Tan

Let $x = \sec\theta$

to get

$\tan\theta$.



were the domain is

$\tan^2 = \sec^2 - 1$

&

$\cot^2 = \csc^2 - 1$

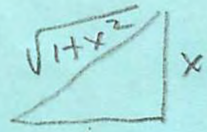
$\sin^2\theta + \cos^2\theta = 1$ leads to:

eg 2

Pg 351

Pg 2

$\int \sqrt{1+x^2}\,dx$



$x = \tan\theta$ ∴ $s^2 + c^2 = 1$

$dx = \sec^2\theta\, d\theta$ $\quad \tau^2 + 1 = \sec^2$

$\int \underset{u}{\sec\theta}\ \underset{dv}{\sec^2\theta\, d\theta}$

$u = \sec\theta \qquad dv = \sec^2\theta\, d\theta$

$du = \sec\theta\tan\theta\, d\theta \qquad v = \tan\theta$

* $\frac{1+\sin\theta}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta}$

$\frac{(1+\sin\theta)^2}{\cos^2\theta} =$

$\left(\frac{1+\sin\theta}{\cos\theta}\right)^2 =$

$(\sec\theta + \tan\theta)^2$

$\int \sec^3\theta\, d\theta = \sec\theta\tan\theta - \int \tan^2\theta\sec\theta\, d\theta$

$= \sec\theta\tan\theta - \int (\sec^2\theta - 1)\sec\theta\, d\theta$

$= \sec\theta\tan\theta + \underline{\int \sec\theta\, d\theta} - \int \sec^3\theta\, d\theta$

$\int \sec\theta\, d\theta$ which we already know...

$2\int \sec^3\theta\, d\theta =$

$\int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\boxed{\int \frac{d\theta}{\cos\theta}}$

$+ \frac{1}{2}\int \frac{\cos\theta\, d\theta}{1-\sin^2\theta}$ $\qquad u = \sin\theta$, $du = \cos\theta\, d\theta$

$+ \frac{1}{2}\int \frac{du}{1-u^2}$ $\qquad = \frac{A}{1-u} + \frac{B}{1+u}$

$1 = A(1+u) + B(1-u)$ $\quad A = \frac{1}{2} \quad B = \frac{1}{2}$

$+ \frac{1}{4}\int \frac{du}{1-u} + \frac{1}{4}\int \frac{du}{1+u}$

$-\frac{1}{4}\ln|1-u| + \frac{1}{4}\ln|1+u|$

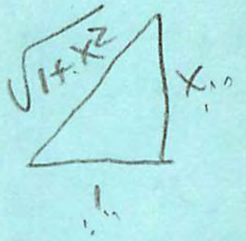
$\int \sec^3\theta\, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{4}\ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right|$ *

$+ \frac{1}{4}\ln|\sec\theta + \tan\theta|^2$

$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|$

more of eg. 2 ↓

$$\int \sec^3\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|$$



using Δ to get back to x

$$= \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ln\left|\sqrt{1+x^2} + x\right|$$

eg 3 $\int \frac{dx}{\sqrt{x^2-9}}$

$s^2 + c^2 = 1$

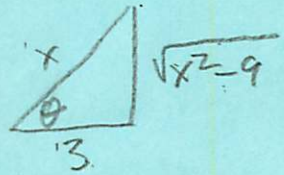
$t^2 + 1 = \sec^2$

$t^2 = \sec^2 - 1$

Let $x = 3\sec\theta \quad \therefore \sec\theta = \frac{x}{3}$

$dx = 3\sec\theta\tan\theta$

$\int \frac{\cancel{3}\sec\theta\cancel{\tan\theta}\,d\theta}{\cancel{3}\cancel{\tan\theta}}$



$\tan\theta = \frac{\sqrt{x^2-9}}{3}$

$3\tan\theta = \sqrt{x^2-9}$

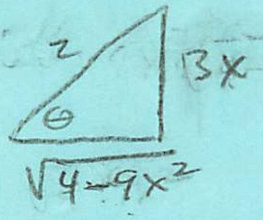
$\int \sec\theta\,d\theta =$ from last example

$= \ln|\sec\theta + \tan\theta|$

$= \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right|$

eg4 $\int \frac{dx}{(4-9x^2)^2}$

if $9x^2 < 4$



$\frac{3x}{2} = \sin\theta$

$\frac{3dx}{2} \cos\theta\,d\theta$

$dx = \frac{2}{3}\cos\theta\,d\theta$

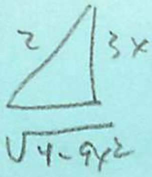
$\frac{\cancel{2}}{3}\int \frac{\cos\theta\,d\theta}{\underset{8}{\cancel{16}}\cos^4\theta} = \frac{1}{24}\int \sec^3\theta\,d\theta$

$= \frac{1}{24}\left[\frac{1}{2}\right]\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right]$

$\frac{1}{48}\left[\frac{2}{\sqrt{4-9x^2}} \cdot \frac{3x}{\sqrt{4-9x^2}} + \ln\left|\frac{2+3x}{\sqrt{4-9x^2}}\right|\right.$

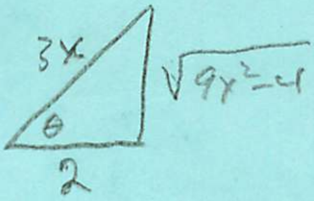
$\frac{1}{48}\left[\frac{6x}{4-9x^2} + \ln\left|\frac{2+3x}{\sqrt{4-9x^2}}\right|\right]$

eg 4 If $9x^2 > 4$ then



is invalid (neg in sq root)

$$\int \frac{dx}{(4-9x^2)^2}$$



$$\int \frac{dx}{\left(-(9x^2-4)\right)^2}$$

$\frac{3x}{2} = \sec\theta$ $\qquad$ $\tan\theta = \frac{\sqrt{9x^2-4}}{2}$

$\frac{3}{2}dx = \sec\theta\tan\theta\, d\theta,$ $\quad 2\tan\theta = \sqrt{9x^2-4}$

$$\frac{2}{3}\int \frac{\sec\theta\tan\theta\, d\theta}{16\tan^4\theta}$$

$$\frac{1}{24}\int \frac{\sec\theta}{\tan^3\theta}\, d\theta$$

$$\frac{1}{24}\int \frac{\cos^3\theta\, d\theta}{\cos\theta\,\sin^3\theta}$$

$$\frac{1}{24}\int \frac{\cos^2\theta\, d\theta}{\sin^3\theta\, d\theta}$$

$$\frac{1}{24}\int \frac{1-\sin^2\theta\, d\theta}{\sin^3\theta}$$

$$\frac{1}{24}\int \csc^3\theta\, d\theta - \int \csc\theta\, d\theta$$

etc. This is enough. end lecture here.

Pg 6066

Eg 5

$$\int \frac{\sqrt[4]{1+x}}{x^2}\,dx = \int \frac{u \cdot 4u^3\,du}{(u^4-1)^2}$$

let

$u = \sqrt[4]{1+x} \Rightarrow x = u^4 - 1$

$4u^3\,du = dx$

$$= 4\int \frac{u^4\,du}{(u^4-1)^2}$$

use partial fractions
To finish if
necessary.

p 350 – 355
Lecture

$s^2+c^2=1$

$t^2+1=\sec^2\theta$

$t^2=\sec^2\theta-1$

eg 3 $\int \frac{dx}{\sqrt{x^2-9}}$

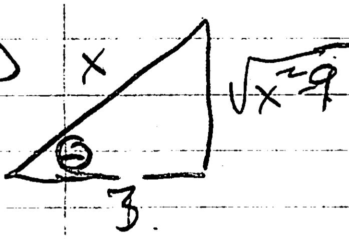
let $x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta$

$\int \frac{3\sec\theta\tan\theta\,d\theta}{3\tan\theta} =$

$\sec\theta = \frac{x}{3}$

$\int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|$



$= \ln\left|\frac{x}{3}+\frac{\sqrt{x^2-9}}{3}\right|$

eg 4 $\int \frac{dx}{(4-9x^2)^2}$ if $9x^2<4$ then

let $9x^2 = 4\sin^2\theta$

$3x = 2\sin\theta$

$dx = \frac{2}{3}\cos\theta\,d\theta$

$\frac{2}{3}\int\frac{\cos\theta\,d\theta}{16(1-\sin^2\theta)^2} = \frac{1}{24}\int\frac{\cos\theta\,d\theta}{(1-\sin^2\theta)^2} = \frac{1}{24}\int\frac{\cos\theta\,d\theta}{\cos^4\theta\,d\theta}$

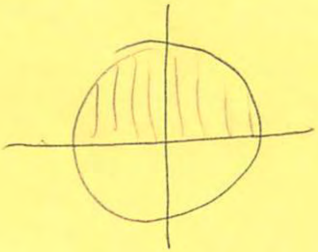
$\frac{1}{24}\int\frac{d\theta}{\cos^3\theta} = \frac{1}{24}\int\sec^3\theta\,d\theta =$

$\frac{1}{24}\left(\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|\right.$

Note if $9x^2>4$ then $9x^2=4\sec^2\theta$

This will "turn around" the items with in the radical so it will be Positive ie so we'll sq root.

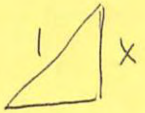
Read through last example. its too hard to find its integral but the method will be used later.

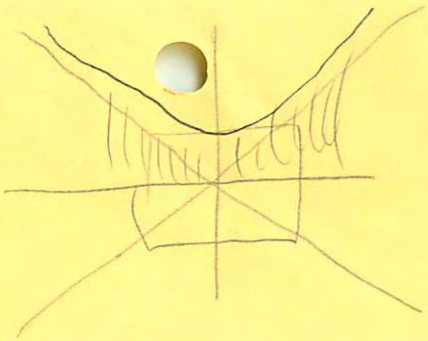


$y = \sqrt{1-x^2}$

$x^2 + y^2 = 1$

$\int \sqrt{1-x^2}\, dx$

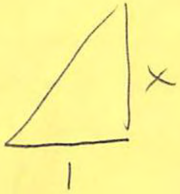


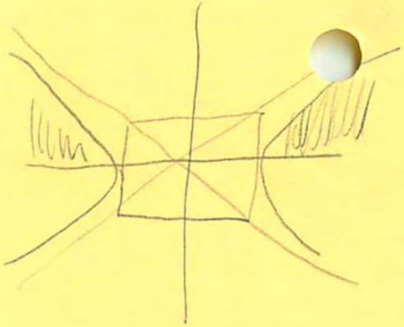


$y = \sqrt{x^2+1}$

$y^2 - x^2 = 1$

$\int \sqrt{x^2+1}\, dx$

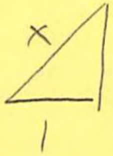




$y = \sqrt{x^2-1}$

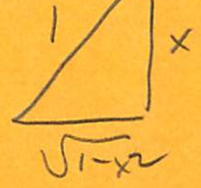
$x^2 - y^2 = 1$

$\int \sqrt{x^2-1}$

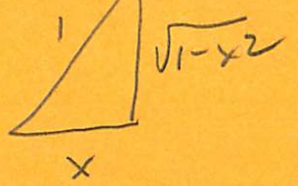


area under which curve?

$$\int \sqrt{1-x^2}\,dx$$



OR



Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$$\sqrt{1-\sin^2\theta} = \cos\theta$$

$$\int \underset{x}{\underset{\downarrow}{\cos\theta}} \cdot \underbrace{\cos\theta\, d\theta}_{dx}$$

$$\int \cos^2\theta\, d\theta$$

$$\frac{1}{2}\int \cos 2\theta + 1\, d\theta$$

Let $x = \cos\theta$

$dx = -\sin\theta\, d\theta$

$$\sqrt{1-\cos^2\theta} = \sin\theta$$

$$-\int \sin\theta \cdot \sin\theta\, d\theta$$

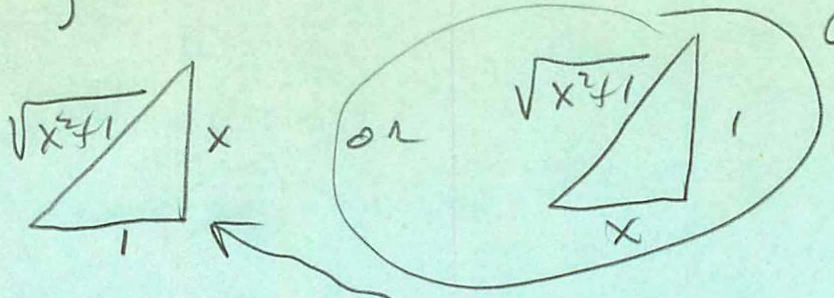
$$-\int \sin^2\theta\, d\theta$$

$$-\frac{1}{2}\int -\cos 2\theta + 1\, d\theta.$$

This choice has the "extra" negative to be concerned about.

$\int \sqrt{x^2+1}\,dx$

on next page.



Now, to get back to x

use Triangle

$= \frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln|\sqrt{x^2+1}+x|$

Let $x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$\sqrt{x^2+1} = \sqrt{\tan^2+1} = \sec\theta$

$\int \sec\theta \cdot \sec^2\theta\,d\theta$

$\int \sec^3\theta\,d\theta =$

$u = \sec\theta \qquad dv = \sec^2\theta\,d\theta$

$du = \sec\theta\tan\theta \qquad v > \tan\theta$

$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$

$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1)\,d\theta$

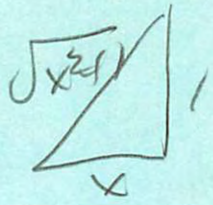
$= \sec\theta\tan\theta - \int \sec^3\theta\,d\theta + \int \sec\theta\,d\theta$

$\sec\theta\tan\theta - \underline{\int \sec^3\theta\,d\theta}_{\hookleftarrow}$

$2\int \sec^3\theta\,d\theta = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|$

$\int \sec^3\theta\,d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|$

Still on $\int \sqrt{x^2+1}\,dx$

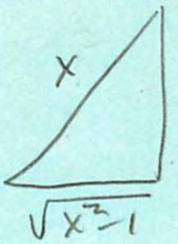
Using 

Let $u = \cot\theta$

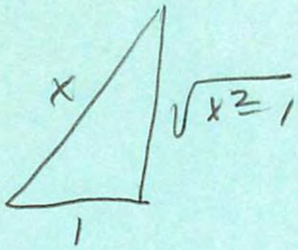
$du = -\cot\theta\csc\theta\,d\theta.$

already we see this is not going in a good direction.

$$\int \sqrt{x^2-1}\ dx$$



or



let $x = \csc\theta$
$dx = -\csc\theta\cot\theta\, d\theta$
Yuk.

let $x = \sec\theta$
$dx = \sec\theta\tan\theta\, d\theta.$

$s^2 + c^2 = 1$
$t^2 + 1 = \sec^2$
$t^2 = \sec^2 - 1$

$$\int \sqrt{\sec^2\theta - 1}\cdot \sec\theta\tan\theta\, d\theta$$

$$\int \sec\theta\tan^2\theta\, d\theta.$$

$$\int \sec\theta(\sec^2\theta - 1)\, d\theta$$

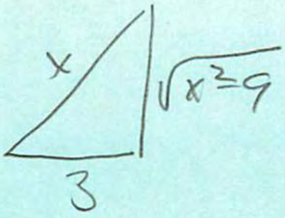
$$\int \sec^3\theta\, d\theta - \int \sec\theta\, d\theta$$

$$\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| - \ln|\sec\theta + \tan\theta|$$

$$\frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta + \tan\theta|$$

$$\frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\ln|x + \sqrt{x^2-1}|$$

$$\int \frac{dx}{\sqrt{x^2-9}}$$



let $\frac{x}{3} = \sec\theta$

$x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\, d\theta$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

$t^2 = \sec^2 - 1$

$$3\int \frac{\sec\theta\tan\theta\, d\theta}{\tan\theta}$$

$$3\int \sec\theta\, d\theta \to 3\ln|\sec\theta + \tan\theta|$$

$$3\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right|$$

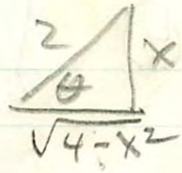
Pg 354

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

① $\int \frac{\sqrt{4-x^2}}{x^3}\,dx$ let $x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$



$\cos\theta = \frac{\sqrt{4-x^2}}{2}$

$\int \frac{2\cos\theta}{8\sin^3\theta} \cdot 2\cos\theta\,d\theta$

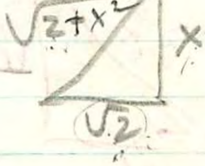
$\frac{1}{2}\int \frac{\cos^2\theta}{\sin^3\theta}\,d\theta$

② $\int \frac{\sqrt{x^2+2}}{4x+1}\,dx$ let $x^2 = 2\tan^2\theta$

$x = \sqrt{2}\tan\theta$

$dx = \boxed{\sqrt{2}\sec^2\theta}$

$\sqrt{2}\sec = \sqrt{\ }$



$\sqrt{2}\int \frac{\sec\theta}{4\sqrt{2}\tan\theta+1} \cdot \sqrt{2}\sec^2\theta\,d\theta$

$2\int \frac{\sec^3\theta}{4\sqrt{2}\tan\theta+1}\,d\theta =$

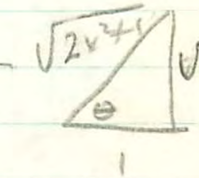
③ $\int \frac{\sqrt{4x^2+2}}{4x+5} =$

$2x^2 = \tan^2\theta$

$\sqrt{2}x = \tan\theta$

$x = \frac{\sqrt{2}}{2}\tan\theta$

$dx = \frac{\sqrt{2}}{2}\sec^2\theta\,d\theta$



$\int \frac{\sqrt{2}\sqrt{2x^2+1}}{2\sqrt{2}\tan\theta+5}\,d\theta$

$\frac{\sqrt{2}}{2}\sqrt{2}\int \frac{\sec^3\theta\,d\theta}{2\sqrt{2}\tan\theta+5}\,d\theta = \int \frac{\sec^3\theta\,d\theta}{2\sqrt{2}\tan\theta+5}$

Pg 354

④ $\int \frac{dx}{\sqrt[4]{x}+\sqrt{x}}$ let $\sqrt{x} = u^4$ $dx = 4u^3\,du$

$\sqrt{x} = u^2$

$\sqrt[4]{x} = u$

$$\int \frac{4u^3\,du}{u+u^2} = \int \frac{4u^2\,du}{1+u}$$

$x = \frac{1}{2}\sec\theta$ $dx = \frac{1}{2}\sec\theta\tan\theta$

⑤ $\int \frac{\sqrt{4x^2-1}}{x+\sqrt{4x^2-1}}\,dx$ let $4x^2 = \sec^2\theta$

$s^2+c^2=1$

$t^2+1 = \sec^2$

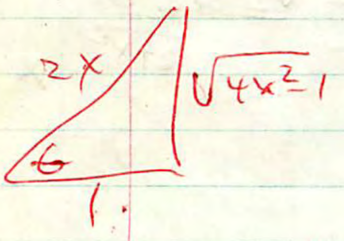
$t^2 = \sec^2 - 1$

$$\frac{1}{2}\int \frac{\tan^2\theta\ \sec\theta\tan\theta}{\frac{1}{2}\sec\theta+\tan\theta}\,d\theta$$

$s^2+c^2=1$

$t^2+1=\sec^2$

$t^2=\sec^2\theta-1$



$\sec\theta = 2x$

$\frac{1}{2}\sec\theta\tan\theta\,d\theta = dx$

p. 354

⑩ $\int \frac{\sqrt[3]{x+8}}{x}\,dx$ let $u^3 = x+8$

$du\ 3u^2 = dx$

$$3\int \frac{u^3\,du}{u^3-8} = 3\int \frac{u^3-8+8\,du}{u^3-8} =$$

$$3\int du + \int \frac{24\,du}{u^3-8} \qquad \frac{A}{u-2} + \frac{Bu+C}{u^2+2u+4}$$

$(u-2)(u^2+2u+4)$

4 4 4

$24 = A(u^2+2u+4) + (Bu+C)(u-2)$

let $u=0$ $24 = 4A \quad -2C$

$u=2$ $24 = 12A \therefore A=2 \therefore C=-8$

$u=1$ $24 = 14 - B + 8$

$24 = 22 - B$

$B = -2$

$$3\int du + \int \frac{2\,du}{u-2} + \int \frac{-2u-8}{u^2+2u+4}$$

$$3u + 2\ln|u-2| + \int \frac{-2u-2}{u^2+2u+4} - \int \frac{6}{u^2+2u+4}\,du$$

$$3u + 2\ln|u-2| - \ln(u^2+2u+4) \rightarrow \int \frac{6\,du}{(u+1)^2+3}$$

$$\frac{1}{\cancel{3}} \int \frac{\overset{2}{\cancel{6}}\,du}{\left(\frac{u+1}{\sqrt{3}}\right)^2+1} = 2\sqrt{3}\,\mathrm{Tan}^{-1}\frac{u+1}{\sqrt{3}}$$ SO NOW FINAL ANSWER

$$3(x+8)^{\frac{1}{3}} + 2\ln|(x+8)^{\frac{1}{3}}-2| - \ln((x+8)^{2/3} + 2(x+8)^{1/3} + 4 +$$

$$-2\sqrt{3}\,\mathrm{Tan}^{-1}\frac{(x+8)^{1/3}+1}{\sqrt{3}}$$

2nd version on next pg.

$s^2+c^2=1$

$t^2+1=\sec^2$

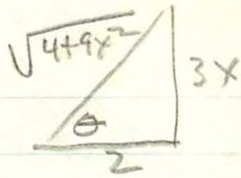
Pr 354

(13) $\int \frac{x^3\,dx}{\sqrt{4+9x^2}}$

let $9x^2 = 4\tan^2\theta$

$3x = 2\tan\theta$

$3\,dx = 2\sec^2\theta\, d\theta$



$\sec = \frac{\sqrt{\quad}}{2}$

$\frac{\cancel{2}}{3}\,\frac{8}{27}\int \frac{\tan^3\theta\,\sec^2\theta\,d\theta}{\cancel{2}\cdot\sec\theta} = \frac{8}{81}\int \tan^3\theta\,\sec\theta\,d\theta$

$\frac{8}{81}\int \tan^2\theta\,\sec\theta\tan\theta\,d\theta = \frac{8}{81}\int(\sec^2\theta-1)\sec\theta\tan\theta\,d\theta$

let $u=\sec\theta$

$du=\sec\theta\tan\theta\,d\theta$

$\frac{8}{81}\int u^2-1\,du$

$\frac{8}{81}\left(\frac{u^3}{3}-u\right) = \frac{8}{243}\left(u^3-3u\right) =$

$\frac{8}{243}\left(\sec^3\theta - 3\sec\theta\right)$

$\sec\theta = \frac{1}{2}\sqrt{4+9x^2}$

$\frac{8}{243}\cdot\left(\frac{(4+9x^2)^{3/2}}{8} - \frac{3}{2}(4+9x)^{\frac{1}{2}}\right) =$

$\frac{4}{243}\left(\frac{1}{4}(4+9x^2)^{3/2} - 3(4+9x)^{1/2}\right) =$

Book ans is algebraic equiv!

$\frac{4(4+9x^2)^{1/2}}{243}\left(\frac{1}{4}(4+9x^2)-3\right)$

#13 different method

↓

Pg 354

⑬ 2nd time $\int \frac{x^3\,dx}{\sqrt{4+9x^2}} = \int \frac{x^2\,x\,dx}{\sqrt{4+9x^2}}$

Let $u = 4+9x^2$

$du = 18x\,dx$

$x^2 = \frac{1}{9}(u-4)$

$$\int \frac{\frac{1}{9}(u-4)\,\frac{1}{18}\,du}{u} =$$

$$\frac{1}{162}\int \frac{u-4}{u^{\frac{1}{2}}}\,du = \frac{1}{162}\int u^{1/2} - \frac{4}{u^{1/2}}\,du =$$

$$\frac{1}{162}\int u^{1/2} - \quad \frac{4}{162}\int \frac{du}{u^{\frac{1}{2}}}$$

$$\frac{2}{3}\cdot\frac{1}{\underset{81}{\cancel{162}}}\,u^{3/2} - \frac{4}{\underset{81}{\cancel{162}}}\,\cancel{2}\;u^{1/2}$$

$$\frac{1}{243}(4+9x^2)^{3/2} - \frac{4}{81}(4+9x^2)^{1/2}$$

$$\frac{1}{243} \qquad - \frac{12}{243}$$

$$\frac{(4+9x^2)^{\frac{1}{2}}}{243}\left[\,4+9x^2 - 12\,\right]$$

Pg 355

⑯ $\int x^3\sqrt{9-x^2}\,dx = \int x^2\sqrt{9-x^2}\,x\,dx$

Let $u = 9-x^2$

$du = -2x\,dx \quad -\frac{1}{2}\int (9-u)\,u^{1/2}\,du$

$-\frac{1}{2}\int 9u^{\frac{1}{2}}\,du + \frac{1}{2}\int u^{3/2}\,du$

$-\frac{1}{\cancel{2}}\cdot\overset{3}{\cancel{9}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot u^{3/2} + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{5}u^{5/2}$

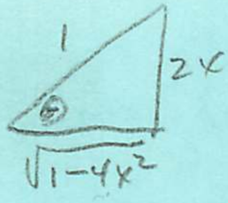
$-3u^{3/2} + \frac{1}{5}u^{5/2}$

$-3(9-x^2)^{3/2} + \frac{1}{5}(9-x^2)^{5/2}$

$(9-x^2)^{\frac{1}{2}}\left(-3(1-x^2) + \frac{1}{5}(81 - 18x^2 + x^4)\right)$

pg 355

(23) $\int \frac{x^3\,dx}{(1-4x^2)^3}$ $4x^2 < 1$



$\sin\theta = 2x$

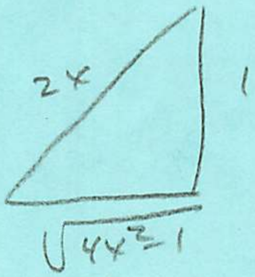
$\cos\theta\,d\theta = 2\,dx$

$x^3 = \frac{\sin^3\theta}{8}$

$\int \frac{\sin^3\theta}{8} \cdot \frac{\cos\theta\,d\theta}{\cos^6\theta} = \frac{1}{8}\int \frac{\sin^3\theta\,d\theta}{\cos^5\theta}$

(24) $\int \frac{x^3\,dx}{(1-4x^2)^3}$ $4x^2 > 1$

$-\int \frac{x^3\,dx}{(4x^2-1)^3}$



$\sin\theta = \frac{1}{2x}$

$2x = \csc\theta$

$2\,dx = -\csc\theta\cot\theta\,d\theta$

$x^3 = \frac{\csc^3\theta}{8}$

$\cot\theta = \sqrt{4x^2+1}$

$-\int \frac{\csc^3\theta}{8} \frac{(-\csc\theta\cot\theta)\,d\theta}{\cot^6\theta}$

$\frac{1}{8}\int \frac{\csc^4\theta}{\cot^5\theta}\,d\theta = \frac{1}{8}\int \frac{\tan^5\theta}{\sin^4\theta}\,d\theta = \frac{1}{8}\int \frac{\sin\theta}{\cos^5\theta}\,d\theta$

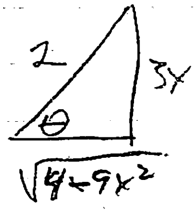
P_r 356

G.Q.

① $\int \frac{x^3\,dx}{1+x^4}$ Let $u=1+x^4$
$du = 4x^3\,dx$

$\frac{1}{4}\int\frac{du}{u} = \frac{1}{4}\ln(1+x^4)$

② $\int\sqrt{4-9x^2}\,dx$ Let $9x^2 = 4\sin^2\theta$
$3x = 2\sin\theta$
$3\,dx = 2\cos\theta\,d\theta$



$\frac{2}{3}\int 2\cos^2\theta\,d\theta = \frac{4}{3}\int\cos^2\theta\,d\theta =$ $2\cos = \sqrt{\ }$

$\frac{4}{3}\int\frac{1}{2}(1+\cos2\theta)\,d\theta = \frac{2}{3}\int d\theta + \frac{2}{3}\int\cos2\theta\,d\theta$

$\frac{2}{3}\theta + \frac{2}{3}\cdot\frac{1}{2}\sin2\theta$

$\frac{2}{3}\theta + \frac{1}{3}\sin2\theta$

$\frac{2}{3}\sin^{-1}\frac{3x}{2} + \frac{2}{3}\sin\theta\cos\theta$

$\frac{2}{3}\sin^{-1}\frac{3x}{2} + \cancel{\frac{2}{3}}\left(\cancel{\frac{3x}{2}}\ \frac{\sqrt{4-9x^2}}{2}\right)$

$\frac{2}{3}\sin^{-1}\frac{3x}{2} + \frac{x}{2}\sqrt{4-9x^2}$

356
GQ

③ $\frac{1}{x^4-1} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{x+1}$

$1 = (Ax+B)(x-1)(x+1) + C(x^2+1)(x+1) + D(x^2+1)(x-1)$

$x=0 \quad 1 = -B + C - D$
$x=1 \quad 1 = 4C$
$x=-1 \quad 1 = -4D$
$x=2 \quad 1 = (2A+B)(3) + 15C + 5D$

~~also could use~~ $\cancel{\frac{1}{2b}\left(\int\frac{1}{a-b} + \int\frac{1}{a+b}\right)}$

$A = 0$
$B = -\frac{1}{2}$
$C = \frac{1}{4}$
$D = -\frac{1}{4}$

$1 = 6A - \frac{3}{2} + \frac{15}{4} - \frac{5}{4}$

$0 = A$

$-\frac{1}{2}\int\frac{dx}{x^2+1}\,dx + \frac{1}{4}\int\frac{dx}{x-1} - \frac{1}{4}\int\frac{dx}{x+1}$

$-\frac{1}{2}\tan^{-1}x + \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1|$

using the standard method for determining the coefficients see Next Page.

↓

③ STd method 356

GQ

$$(Ax+B)(x^2-1)+C(x^2+1)(x+1)+D(x-1)(x^2+1)$$

$$\begin{array}{llll} Ax^3 & +Bx^2 & -Ax & -B \\ Cx^3 & +Cx^2 & +Cx & +C \\ Dx^3 & -Dx^2 & +Dx & -D \\ \hline (A+C+D)x^3 & & & \end{array}$$

$A+C+D=0$

$B+C-D=0$

$-A+C+D=0$

$-B+C-D=1$

$A+B+2C=0$

$A-B+2C=1$

$-A-B+2C=1$

$0+B+\frac{1}{2}=0$

$B=-\frac{1}{2}$

$2A+4C=1$

$4C=1 \quad C=\frac{1}{4}$

$2A+1=1$

$A=0$

$A+C+D=0$

$0+\frac{1}{4}+D=0$

$D=-\frac{1}{4}$

Pg 356
GQ

④ $\int \tan^5 2x \underline{\sec^2 2x\, dx}$ $\quad u = \tan 2x$

$du = 2\sec^2 2x\, dx$

$\frac{1}{2}\int u^5\, du = \frac{1}{12}u^6 = \frac{1}{12}\tan^6 2x$

⑤ $\int \frac{x^4}{x^4-1}\,dx = \int 1\,dx + \int \frac{1}{x^4-1}\,dx$

$s+c=1$ usig Pblm 3

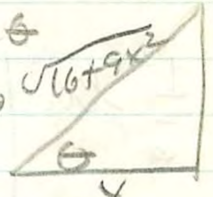
$t+1=\sec$

$x + \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x$

⑥ $\int \frac{dx}{\sqrt{9x^2+16}}$ $\quad$ let $9x^2 = 16\tan\theta$

$3x = 4\tan\theta$

$3\,dx = 4\sec^2\theta\, d\theta$



$\frac{1}{4}\,\frac{4}{3}\int \frac{\sec^2\theta\, d\theta}{\sec\theta}$

$4\sec = \sqrt{}$

changebacktoX.

$\frac{1}{3}\int \sec\theta\, d\theta = \frac{1}{3}\ln\left|\frac{1+\sin\theta}{1-\sin\theta}\right|$ usig resultTab eg 7 pg 348-349

[illegible]

⑦ $\int \frac{dx}{2\sqrt{x}-\sqrt[4]{x}}$ $\quad$ let $u^4 = x$

$4u^3\,du = dx$

$$\begin{array}{r} 2u+1 \\ 2u-1\overline{)\,4u^2} \\ 4u^2-2u \\ \hline +2u \\ 2u \end{array}$$

$\int \frac{4u^3\,du}{2u^2-u} = \int \frac{4u^2\,du}{2u-1}$

$\int 2u+1\,du + \int \frac{du}{2u-1} = u^2+u+\frac{1}{2}\ln|2u-1|$

$\sqrt{x} + \sqrt[4]{x} + \frac{1}{2}\ln|2\sqrt[4]{x}-1|$

P357

⑧ $\int \frac{dx}{(x-3)(\sqrt{x+3})}$ let $u^2 = x+3$

$2u\,du = dx$

$\int \frac{2\cancel{u}\,du}{(u^2-6)\cancel{u}} = \int \frac{2\,du}{u^2-6}$ $\frac{A}{u+\sqrt6} + \frac{B}{u-\sqrt6}$

$2 = A(u-\sqrt6) + B(u+\sqrt6)$

$u=\sqrt6 = \cancel{2} = \cancel{2}B\sqrt6$ $B = \frac{1}{\sqrt6}$

$u=-\sqrt6 = \cancel{2} = -\cancel{2}A\sqrt6$ $A = -\frac{1}{\sqrt6}$

$-\frac{1}{\sqrt6}\int \frac{du}{u+\sqrt6} + \frac{1}{\sqrt6}\int \frac{du}{u-\sqrt6} = \ln\left|\frac{u-\sqrt6}{u+\sqrt6}\right|$

$\frac{1}{\sqrt6} \ln\left|\frac{\sqrt{x+3}-\sqrt6}{\sqrt{x+3}+\sqrt6}\right|$

⑨ $\int \frac{dx}{3-x^2}$ Same as part of 8. ∴ skip.

#8 $\int dx$ use triangle much cleaner.

Pg 357
GQ

⑩ $\int \sin^5 2x\,dx$ = use formula #46, #45

$-\frac{1}{10}\sin^4 2x\cos 2x + \frac{4}{5}\int \sin^3 2x\,dx$

$-\frac{1}{10}\sin^4 2x\cos 2x + \frac{4}{5}\left[-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x\right]$

$-\frac{1}{10}\sin^4 2x\cos 2x - \frac{4}{10}\cos 2x + \frac{2}{15}\cos^3 2x$

⑩ agn using substitution

$\int \sin^5 2x\,dx = \int \sin^4 2x\,\underline{\sin\,dx}$ $\longrightarrow -du = \sin 2x$

$= \int (1-\cos^2 2x)^2 \sin 2x\,dx$ Let $u = \cos 2x$
$du = -2\sin 2x$

$-\frac{1}{2}\int 1 - 2u^2 + u^4\,du$

$-\frac{1}{2}\left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right] =$

$-\frac{1}{2}\cos 2x + \frac{1}{3}\cos^3 2x - \frac{1}{10}\cos^5 2x.$

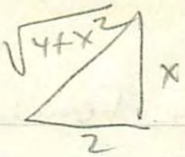
⑪ $\int e^x \cos 2x\,dx$ $u = e^x$ $v = \frac{1}{2}\sin 2x$
$du = e^x dx$ $dv = \cos 2x\,dx$

$= \frac{1}{2}e^x \sin 2x - \frac{1}{2}\int e^x \sin 2x\,dx$
$u = e^x$ $v = -\frac{1}{2}\cos 2x$
$du = e^x dx$ $dv = +\sin 2x\,dx$

$= \frac{1}{2}e^x \sin 2x - \frac{1}{2}\left[\frac{e^x}{2}\cos 2x + \frac{1}{2}\int e^x \cos 2x\,dx\right] = \frac{1}{4}$

$= \frac{4}{5}\left(\frac{1}{2}e^x \sin 2x + \frac{1}{4}e^x \cos 2x\right] = \frac{2}{5}e^x \sin 2x + \frac{1}{5}e^x \cos 2x$

#14



$2\tan\theta = x$

$2\sec^2\theta\, d\theta = dx$

$2\sec\theta = \sqrt{4+x^2}$

$\int \frac{dx}{x\sqrt{4+x^2}}$

$\frac{1}{2}\int \frac{\cancel{2}\sec^{\cancel{2}}\theta\, d\theta}{\cancel{2}\,\cancel{\sec\theta}\tan\theta} = \frac{1}{2}\int \frac{\sec\theta\, d\theta}{\tan\theta} = \frac{1}{2}\int \frac{\frac{1}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}\, d\theta$

$\frac{1}{2}\int \frac{d\theta}{\sin\theta} = \frac{1}{2}\int \frac{\sin\theta\, d\theta}{1-\cos^2\theta}$ $\quad u = \cos\theta$, $du = -\sin\theta\, d\theta$

$= -\frac{1}{2}\int \frac{du}{1-u^2}$ $\quad \frac{A}{1-u} + \frac{B}{1+u}$

$1 = A(1+u) + B(1-u)$

$1 \quad 1 = 2A \quad A = \frac{1}{2}$

$-1 \quad 1 = 2B \quad B = \frac{1}{2}$

$\frac{1}{2}\left(-\frac{1}{2}\right)\int \frac{du}{1-u} - \frac{1}{4}\int \frac{du}{1+u}$ $\rightarrow$ $\frac{1}{2}\ln\left|-\frac{2-\sqrt{4+x^2}}{x}\right|$

$\frac{1}{2}\ln\left|\frac{\sqrt{4+x^2}-2}{x}\right|$

$+\frac{1}{4}\ln|1-u| - \frac{1}{4}\ln|1+u|$

$\frac{1}{4}\ln\left|\frac{1-u}{1+u}\right|$

$\frac{1}{4}\ln\left|\frac{1-\cos x}{1+\cos x}\right|$

$\frac{1}{4}\ln\left|\frac{2-\sqrt{4+x^2}}{2+\sqrt{4+x^2}}\right|$

$\frac{1}{4}\ln\left|\frac{4-4\sqrt{4+x^2}+4+x^2}{4-(4+x^2)}\right|$

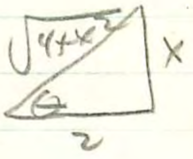
$\frac{1}{4}\ln\left|\frac{\quad}{-x^2}\right|$

Pg 357
GQ

⑬ $\int \frac{x\,dx}{1+x^4}$ let $u = x^2$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}u = \frac{1}{2}\tan^{-1}x^2$

⑭ $\int \frac{dx}{x\sqrt{4+x^2}}$ let $x^2 = 4\tan^2\theta$
$x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$



Somethings wrong in this!

$\int \frac{2\sec^2\theta\,d\theta}{2\tan\theta\,2\sec\theta} = \frac{1}{2}\int \frac{\sec\theta\,d\theta}{\tan\theta} =$ $\cos\ \frac{2}{\sqrt{\ }}$

$\frac{\frac{1}{\cos}}{\frac{\sin}{\cos}}$

$\frac{1}{2}\int \frac{d\theta}{\sin\theta}$

$\frac{1}{2}\int \frac{\sin\theta\,d\theta}{\sin^2\theta} = \frac{1}{2}\int \frac{\sin\theta\,d\theta}{1-\cos^2\theta}$ let $u = \cos\theta$
$du = -\sin\theta\,d\theta$

$-\frac{1}{2}\int \frac{du}{1-u^2} = -\frac{1}{2}\left[\frac{1}{2}\int \frac{du}{1-u} - \int \frac{du}{1+u}\right]$

$-\frac{1}{4}\ln\left|\frac{1-u}{1+u}\right| = -\frac{1}{4}\ln\left|\frac{1-\cos\theta}{1+\cos\theta}\right| =$

$-\frac{1}{4}\ln\left|\frac{1-\frac{2}{\sqrt{4+x^2}}}{1+\frac{2}{\sqrt{4+x^2}}}\right| = -\frac{1}{4}\ln\left|\frac{\sqrt{4+x^2}-2}{\sqrt{4+x^2}+2}\right|$

Pg 357
Review
Ex

① A) $\int_0^{\frac{\pi}{2}} \sqrt{(1+\cos\theta)^3}\,\sin\theta\, d\theta$

let $u = \cos\theta$
$du = -\sin\theta\, d\theta$

$-\int_1^0 \sqrt{(1+u)^3}\, du$

$\theta = 0 \Rightarrow u = 1$
$\theta = \frac{\pi}{2} \Rightarrow u = 0$

OR (since this still isn't too simple

let $u = 1+\cos\theta$
$du = -\sin\theta$
$\theta = 0 \Rightarrow u = 2$
$\theta = \frac{\pi}{2} \Rightarrow u = 1$

$\int_1^2 u^{3/2}\, du$

(notice change of direction on limits of integration

$\sqrt{32}$
$4\sqrt{2}$

b) $\frac{2}{5}u^{5/2}\Big|_1^2 = \frac{2(2)^{5/2}}{5} - \frac{2}{5} = \frac{2}{5}(4\sqrt{2}-1)$

② a) $\int \ln x\, dx$

$u = \ln x \quad v = x$
$du = \frac{1}{x}dx \quad dv = dx$

$x\ln x - \int dx = x\ln x - x$

b) $\int \frac{\ln x}{x}\, dx$

let $u = \ln x$
$du = \frac{1}{x}\, dx$

$\int u\, du = \frac{1}{2}u^2 \Rightarrow \frac{1}{2}(\ln x)^2$

c) $\int \frac{dx}{\ln x}$ This must be the non elementary fctn since the 1st two worked.

357

$$7\overline{)127}\quad 18,\ 7,\ 57$$

③ A) $\int_1^2 (1+x^3)^2\,dx = \int_1^2 1+2x^3+x^6\,dx =$

$x+\frac{1}{2}x^4+\frac{1}{7}x^7\Big|_1^2 = 2+8+\frac{128}{7}-1-\frac{1}{2}-\frac{1}{7}$

$8\frac{1}{2}+\frac{127}{7}$ $\quad \frac{17}{2}+\frac{127}{7}$

$\frac{119+254}{14} = \frac{373}{14}$

B) $\int_1^2 (1+x^3)^2 x^2\,dx$ $\quad$ let $u = (1+x^3)$ $\quad x=1 \Rightarrow u=2$

$du = 3x^2\,dx$ $\quad x=2 \Rightarrow u=9$

$\frac{1}{3}\int_2^9 u^2\,du = \frac{1}{9}u^3\Big|_2^9 = 81-\frac{8}{9} = \frac{721}{9}$

④ A) $\int \frac{e^x\,dx}{5e^{2x}-3}$ not in tables as it stands so we need to make a few changes.

let $u = e^x$

$du = e^x\,dx$

#27 $\int \frac{du}{5u^2-3} = \frac{1}{2\sqrt{15}} \ln\left|\frac{x\sqrt{5}-\sqrt{3}}{x\sqrt{5}+\sqrt{3}}\right|$

B) # $\int \frac{dx}{\sqrt{x^2-3}} = \int \frac{dx}{\sqrt{x^2-(\sqrt{3})^2}}$

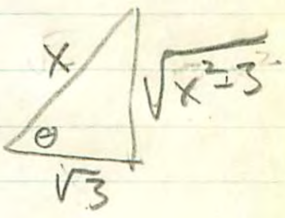
$s^2+c^2=1$

$t^2+1=\sec^2$

let $3\sec^2\theta = x^2$

$\sqrt{3}\sec\theta = x$

$dx = \sqrt{3}\sec\theta\tan\theta\,d\theta$



$\sqrt{3}\int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{3}\tan\theta\,d\theta}$

#7 $\int \sec\theta\,d\theta = \ln|\sec x+\tan x|$

use 33 directly.

$S^2+C^2=1$ Pg 359

$T^2+1=\sec^2$

⑤ a) $\int \frac{dx}{x^3} = -\frac{1}{2x^2}$

b) $\int \frac{dx}{\sqrt{x+1}}$ Let $u^2 = x+1$

$2u\,du = dx$

$$\int \frac{2u\,du}{u} = \int 2\,du = 2u = 2\sqrt{x+1}$$

c) $\int \frac{e^x\,dx}{1+5e^x}$ Let $u = 1+5e^x$

$du = 5e^x\,dx$

$$\frac{1}{5}\int \frac{du}{u} = \frac{1}{5}\ln|1+5e^x|$$

⑥ $\int \frac{5x^4-5x^3+10x^2-8x+4}{(x^2+1)(x-1)}$

$$\begin{array}{r|l} & 5x \\ \hline x^3-x^2+x-1 & 5x^4-5x^3+10x^2-8x+4 \\ & 5x^4-5x^3+5x^2-5x \\ \hline & 5x^2-3x+4 \end{array}$$

$$\int 5x\,dx + \int \frac{5x^2-3x+4}{(x^2+1)(x-1)}$$

$5x^2-3x+4 = (Ax+B)(x-1)+C(x^2+1)$

$x=0$ $4 = -B + C$ $B=-1$

$x=1$ $6 = 2C$ $\therefore C=3$

$x=-1$ $12 = -2(-A+B)+2C$

$12 = 2A-2B+6$

$6 = 2A-2B$

$3 = A-B$

$3 = A+1$ $A=2$

$$\int 5x\,dx + \int \frac{2x-1}{x^2+1}dx + \int \frac{3\,dx}{x-1}$$

$$\frac{5x^2}{2} + 3\ln|x-1| + \int \frac{2x-1}{x^2+1}dx$$

$$+\int \frac{du}{u} - \int \frac{dx}{x^2+1}$$

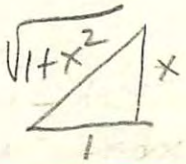
$$\frac{5x^2}{2} + 3\ln|x+1| + \ln|x^2+1| - \tan^{-1}x$$

$u = x^2+1$

$du = 2x\,dx$

$$\frac{2x}{x^2+1} - \frac{1}{x^2+1}$$

#7 Pg 357



$$\int \frac{x^3\,dx}{(1+x^2)^4}$$

$$\int \frac{\tan^3\theta \sec^2\theta\, d\theta}{\sec^8\theta}$$

$$\int \sin^3\theta \cos^3\theta\, d\theta$$ let $u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\int \sin^3\theta\,(1-\sin^2\theta)\cos\theta\, d\theta$$

$$\int u^3(1-u^2)\,du$$

$$3x^4+x^6 \overset{?}{=} -3x^2-1$$

$$x^4(3+x^2) = -(3x^2+1)$$

$$\int u^3 - u^5\,du$$

$$\frac{u^4}{4} - \frac{u^6}{6}$$

$$\frac{\sin^4\theta}{4} - \frac{\sin^6\theta}{6}$$

$$\frac{1}{12}\left[3(1-\cos^2\theta)^2 - 2(1-\cos^2\theta)^3\right]$$

$$\frac{1}{12}(1-\cos^2\theta)^2\left[3-2(1-\cos^2\theta)\right]$$

$$\frac{1}{12}(1-\cos^2\theta)^2\left[1+2\cos^2\theta\right]$$

$$\frac{1}{12}\left(1-\frac{1}{1+x^2}\right)^2\left[1+\frac{2}{1+x^2}\right]$$

$$\frac{1}{12}\left(\frac{x^4}{(1+x^2)^2}\right)\left(\frac{1+x^2+2}{1+x^2}\right]$$

$$\frac{3x^4+x^6}{12(1+x^2)^3}$$

used as Quiz pblm

Let $u = 1 + x^2$
$du = 2x\,dx$
$x^2 = u - 1$

Pg 357

also on previous page

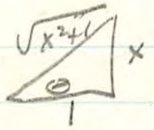
⑦ $\int \frac{x^3 dx}{(1+x^2)^4}$

A) $\frac{1}{2}\int \frac{u-1}{u^4}\,du = \frac{1}{2}\int \frac{du}{u^3} - \frac{1}{2}\int \frac{du}{u^4}$

$-\frac{1}{4u^2} + \frac{1}{6u^3} = \frac{-3u+2}{12u^3} =$

$\frac{-3-3x^2+2}{12(1+x^2)^3} = -\frac{3x^2+1}{12(1+x^2)^3}$

B) Let $x = \tan\theta$
$dx = \sec^2\theta\,d\theta$



$\int \frac{\tan^3\theta \sec^2\theta\,d\theta}{\sec^8\theta} = \int \frac{\tan^2\theta \sec\theta\tan\theta}{\sec^7\theta}\,d\theta$

$s^2 + c^2 = 1$
$t^2 + 1 = \sec^2$

$\int \frac{(\sec^2\theta - 1)\sec\theta\tan\theta\,d\theta}{\sec^7\theta} = \int \frac{\sec\theta\tan\theta\,d\theta}{\sec^5\theta} - \int \frac{\sec\theta\tan\theta\,d\theta}{\sec^7\theta}$

Let $u = \sec\theta$
$du = \sec\theta\tan\theta$

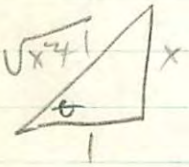
$\int \frac{du}{u^5} - \int \frac{du}{u^7} = -\frac{1}{4u^4} + \frac{1}{6u^6} =$

$\frac{-3u^2+2}{12u^6} = \frac{-3\sec^2\theta + 2}{12\sec^6\theta} = \frac{-3(x^2+1)+2}{12(x^2+1)^3} =$

$\frac{-3x^2-1}{12(x^2+1)^3}$

OR

$\int \frac{\tan^3 \sec^2\theta\,d\theta}{\sec^8\theta} = \int \frac{\tan^3}{\sec^6\theta}\,d\theta =$



$1 - \cos s \quad 1 - s^2$

$\int \frac{\sin^3\theta \cos^6\theta\,d\theta}{\cos^3\theta} = \int \sin^3 \cos^3\theta\,d\theta$

Let $u = \cos\theta$
$du = -\sin\theta\,d\theta$

$-\int (1-\cos^2\theta)\cos^3\theta \sin\theta\,d\theta$

$\int u^3(-1+u^2)\,du = -\int u^3 du + \int u^5 du =$

$-\frac{1}{4}u^4 + \frac{1}{6}u^6 = -\frac{1}{4}\cos^4\theta - \frac{1}{6}\cos^6\theta$

$\cos\theta = \frac{1}{\sqrt{x^2+1}}$

$-\frac{1}{4}\frac{1}{(x^2+1)^2} + \frac{1}{6}\frac{1}{(x^2+1)^3} = \frac{-3(x^2+1)+2}{12(x^2+1)^3}$

357

⑧ $\int_0^3 \frac{x^3\,dx}{\sqrt{x+1}}$ let $u = x+1$, $du = dx$ $x=0 \Rightarrow u = 1$, $x = 3 \Rightarrow u = 4$

$\int_1^4 \frac{(u-1)^3\,du}{u^{\frac{1}{2}}} = \int_1^4 \frac{u^3 - 3u^2 + 3u - 1}{u^{\frac{1}{2}}}\,du = \int_1^4 u^{5/2} - 3u^{3/2} + 3u^{1/2} - \frac{1}{u^{1/2}}\,du$

$\frac{2}{7}u^{7/2} - \frac{6}{5}u^{5/2} + 2u^{3/2} - 2u^{1/2}\Big|_1^4$

$\frac{256}{7} - \frac{192}{5} + 16 - 4 - \frac{2}{7} + \frac{6}{5} - 2 + 2$

$\frac{254}{7} - \frac{186}{5} + 12$

$\frac{1270 - 1302 + 420}{35} = \frac{388}{35}$

⑨ $\int x^2 \ln(1+x)\,dx$ let $u = \ln(1+x)$ $v = \frac{1}{3}x^3$
$du = \frac{1}{1+x}$ $dv = x^2$

$\frac{1}{3}x^3 \ln(1+x) - \frac{1}{3}\int \frac{x^3}{1+x}$

$$\begin{array}{r} x^2 - x + 1 \\ x+1\,\big)\overline{x^3\qquad\quad} \\ \underline{x^3 + x^2} \\ -x^2 \\ \underline{-x^2 - x} \\ x \\ \underline{x+1} \\ -1 \end{array}$$

$\frac{1}{3}x^3 \ln(1+x) - \frac{1}{3}\left[\int \left(x^2 - x + 1 - \frac{1}{x+1}\right)dx\right]$

$\frac{1}{3}x^3 \ln(1+x) - \frac{1}{9}x^3 + \frac{1}{6}x^2 - \frac{1}{3}x + \frac{1}{3}\ln(x+1)$

Book answer differs by a constant $\left(-\frac{11}{18}\right)$.

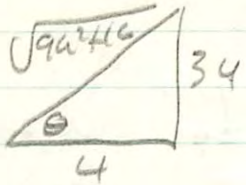
Pg 357

(10) $\int \frac{x\,dx}{\sqrt{9x^4+16}}$ let $u = x^2$, $du = 2x\,dx$ $\frac{1}{2}\int \frac{du}{\sqrt{9u^2+16}}$

let $9u^2 = 16\tan^2\theta$

$3u = 4\tan\theta$

$3\,du = 4\sec^2\theta\,d\theta$



$4\sec\theta = \sqrt{}$

$\frac{\cancel{4}}{3}\cdot\frac{1}{2}\int \frac{\sec^2\theta\,d\theta}{\cancel{4}\sec\theta\,d\theta} = \frac{1}{6}\int \sec\theta\,d\theta = \frac{1}{6}\ln|\sec\theta+\tan\theta| =$

$\frac{1}{6}\ln\left(\frac{\sqrt{9x^4+16}}{4} + \frac{3x^2}{4}\right) = \frac{1}{6}\ln\left(\sqrt{9x^4+16}+3x^2\right) \underline{-\frac{1}{6}\ln 4}$

(b) using tables, we still need the 1st substitution.

so $u = x^2$ we cannot use #38. why not?

$du = 2x\,dx$ $a \neq 1$.

$\frac{1}{2}\int \frac{du}{\sqrt{9u^2+16}}$ Using #40 $a = 9, b=0, c=16$

$\frac{1}{2}\cdot\frac{1}{3}\ln|2\cdot 9\cdot u + 2\cdot 3\sqrt{9u^2+16}|$

Notice these differ by a constant

$\frac{1}{6}\ln|18u+6\sqrt{9u^2+16}|$

$\frac{1}{6}\ln|18x^2+6\sqrt{9x^4+16}|$

$\frac{1}{6}\ln 6|3x^2+\sqrt{9x^4+16}| =$

$\frac{1}{6}\ln|3x^2+\sqrt{9x^4+16}| \underline{+\frac{1}{6}\ln 6}$

#11

$$\int \frac{\sin\theta}{1+\sin^2\theta}\,d\theta$$

$$\int \frac{\sin\theta\,d\theta}{1+1-\cos^2\theta} \qquad \begin{array}{l} u = \cos\theta \\ du = -\sin\theta\,d\theta \end{array}$$

$$-\int \frac{du}{2-u^2} = \int \frac{du}{u^2-2} \qquad \frac{A}{u-\sqrt{2}} + \frac{B}{u+\sqrt{2}}$$

$$A, b = \frac{1}{2}$$

$$\frac{1}{2}\int \frac{du}{u-\sqrt{2}} + \frac{1}{2}\int \frac{du}{u+\sqrt{2}}$$

$$\frac{1}{2}\ln|u^2-\sqrt{2}|$$

$$\frac{1}{2}\ln|\cos^2\theta-2|$$

too much work.

Do it This way

⑪ A) $\int \frac{\sin\theta \, d\theta}{1+\sin^2\theta}$ using $\tan\frac{\theta}{2} = u$ $d\theta = \frac{2\,du}{1+u^2}$

$\cos\theta = \frac{1-u^2}{1+u^2}$, $\sin\theta = \frac{2u}{1+u^2}$

$$\int \frac{\frac{2u}{1+u^2} \cdot \frac{2\,du}{1+u^2}}{1 + \frac{2u}{1+u^2}} = \int \frac{\frac{4u\,du}{(1+u^2)^2}}{\frac{1+u^2+2u}{1+u^2}} = \int \frac{\frac{4u\,du}{1+u^2}}{(u+1)^2} =$$

$$\int \frac{4u(u+1)^2\,du}{1+u^2} = \int \frac{4u^3+8u^2+4u}{u^2+1}\,du$$

$$\begin{array}{r} 4u+4 \\ u^2+1 \overline{)\,4u^3+8u^2+4u} \\ \underline{4u^3+4u^2} \\ 4u^2+4u \\ \underline{4u^2+\ \ 4} \\ 4u-4 \end{array}$$

$$\int 4u+4 + \frac{4u-4}{u^2+1}\,du$$

$$2u^2+4u+2\int\frac{2u\,du}{u^2+1} - 4\int\frac{du}{u^2+1}$$

$$2u^2+4u+2\ln(u^2+1) - 4\tan^{-1}u =$$

$$2\tan^2\frac{\theta}{2} + 4\tan\frac{\theta}{2} + 2\ln(\sec^2\frac{\theta}{2}) - 4\tan^{-1}(\tan\frac{\theta}{2})$$

$$2\tan^2\frac{\theta}{2} + 4\tan\frac{\theta}{2} + 4\ln\sec\frac{\theta}{2} - 2\theta.$$

# 10

11 B) $\int \frac{\sin\theta\,d\theta}{2-\cos^2\theta}$ let $u = \cos\theta$, $du = -\sin\theta\,d\theta$ $-\int\frac{du}{2-u^2} =$

$-\sin^{-1}\frac{u}{\sqrt{2}}$

1
1 1
1 2 1
1 3 3 1
1 4 6

This is (A)
A only.

⑫ $\int \sin^5\theta\, d\theta = \int \sin\theta\,(1-\cos^2\theta)^2\, d\theta =$

let $u = \cos\theta$
$du = -\sin\theta$

$-\int (1-u^2)^2\, d\theta = -\int 1 - 2u^2 + u^4\, du$

$-u + \frac{2}{3}u^3 - \frac{1}{5}u^5 = -\cos\theta + \frac{2}{3}\cos^3\theta - \frac{1}{5}\cos^5\theta$

$s^2 + c^2 = 1$
$t^2 + 1 = \sec^2$

$\int \tan^6\theta\, d\theta = \int \tan^4\theta\, \tan^2\theta\, d\theta =$

$-\int \tan^4\theta\, d\theta + \int \underset{u}{\tan}^4 \boxed{\sec^2\theta\, d\theta} =$

$-\int \tan^2\theta\, \tan^2\theta\, d\theta + \int \tan^4 \sec^2\theta\, d\theta$

$-\int \tan^2\theta\, (\sec^2\theta - 1)\, d\theta +$

$\int \tan^2\theta\, d\theta - \int \tan^2\theta \sec^2\theta\, d\theta + \int \tan^4 \sec^2\theta\, d\theta$

$\int (\sec^2\theta - 1)\, d\theta - \int \qquad + \int$

$\int \sec^2\theta\, d\theta - \int d\theta - \int \tan^2 \sec^2\theta\, d\theta + \int \tan^4 \sec^2\theta\, d\theta$

$u = \tan\theta \quad du = \sec^2\theta\, d\theta \quad u = \tan\theta \quad du = \sec^2\theta\, d\theta$

$\tan\theta - \theta - \int u^2\, du + \int u^4\, du$

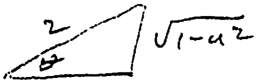
$\tan\theta - \theta - \frac{1}{3}u^3 + \frac{1}{8}u^5 =$

$\tan\theta - \theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta.$

$2u = -8\sin\theta\cos\theta$ $\quad u^2 = 1 - 4\sin^2\theta$

$\sin^2\theta = \frac{1}{4}\; 1-u^2$

$\sin\theta = \frac{1}{2}\sqrt{1-u^2}$



pg 357

⑬ a) not elementary

b) $\int\sqrt{4-4\sin^2\theta}\,d\theta = 2\int\sqrt{1-\sin^2\theta}\,d\theta =$

$2\int\cos\theta\,d\theta = 2\sin\theta$

c) $\int\sqrt{1+\cos\theta}\,d\theta\;\frac{\sqrt{1-\cos\theta}}{\sqrt{1-\cos\theta}} = \int\frac{\sqrt{1-\cos^2\theta}}{\sqrt{1-\cos\theta}}\,d\theta =$

$\int\frac{\sin\theta\,d\theta}{\sqrt{1-\cos\theta}}$ $\quad$ Let $u^2 = 1-\cos\theta$

$2u\,du = \sin\theta\,d\theta$

$\int\frac{2u\,du}{u} = 2\int du = 2u = 2\sqrt{1-\cos\theta}$

⑭ A) $\int_{-1}^{4}\frac{\overset{u-3}{x}+2\,dx}{\sqrt{x+3}}$ $\quad$ Let $u = x+3$ $\quad x=-1 \Rightarrow u=-4$

$du = dx$ $\quad x = 4 \Rightarrow u = 1$

$u-3 = x$

this is enough. →

$\int_{-4}^{1}\frac{u+1\,du}{u^{\frac{1}{2}}} = \int_{2}^{7}u^{\frac{1}{2}} + u^{-\frac{1}{2}}\,du =$

$\frac{2}{3}u^{3/2} - u^{\frac{1}{2}}\Big|_{2}^{7} =$

$u^{\frac{1}{2}}\left(\frac{2}{3}u-1\right)$ $\quad \sqrt{7}\left(\frac{14}{3}-\frac{3}{3}\right) - \sqrt{2}\left(\frac{4}{3}-1\right)$

Pg 358

Skip

⑮ A) $\int_0^1 \sqrt[3]{x}\,\sqrt{x}\,dx = \int_0^1 x^{\frac{1}{3}}\cdot x^{\frac{1}{2}}\,dx = \int_0^1 x^{\frac{5}{6}}\,dx =$

$\frac{6}{11}x^{\frac{11}{6}}\Big|_0^1 = \frac{6}{11}$

B) $\int_0^1 \sqrt[3]{1-x}\,\sqrt{x}\,dx$

⑯ Skip

⑰ $\frac{2x^2+3x+1}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

-2

$2x^2+3x+1 = A(x^2+x+1)+(Bx+C)(x-1)$

$x=0 \qquad 1 = A - C$

$x=1 \qquad 6 = 3A \quad \therefore A = 2 \quad C = 1$

$x=-1 \qquad 0 = A + 2B - 2C$

$0 = \cancel{2} + 2B - \cancel{2} \quad B = 0$

$= \frac{2}{x-1} + \frac{1}{x^2+x+1}$

⑱ $\frac{x^4+2x^2-2x+2}{x^3-1}$

$$\begin{array}{r} x \\ x^3+1 \overline{\big)\, x^4+2x^2-2x+2} \\ \underline{x^4 \qquad \pm x} \\ 2x^2-x+2 \end{array}$$

$x + \frac{2x^2-x+2}{(x-1)(x^2+x+1)}$

$2x^2-x+2 = A(x^2+x+1)+(Bx+C)(x-1)$

$x=0 \qquad 2 = A - C$

$x=1 \qquad 3 = 3A \Rightarrow A = 1 \quad C = -1$

$x=2 \qquad 8 = \cancel{7A} + 2B + \cancel{C} \Rightarrow B = 1$

$2 = 2B$

$B = 1$

$x + \frac{1}{x-1} + \frac{x-1}{x^2+x+1}$

Pg 358

(19) $\frac{2x-1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$

$2x-1 = A(x^2-x+1) + (Bx+C)(x+1)$

$x=-1 \quad -3 = 3A \Rightarrow A = -1$

$x=0 \quad -1 = A + C \therefore C = 0$

$x=1 \quad 1 = A + 2B + 2C$

$1 = -1 + 2B \Rightarrow B = 1$

$\frac{-1}{x+1} + \frac{x}{x^2-x+1}$

used on Quiz pblm

(20)
$$x^4-1 \overline{\smash{)}\,x^4+3x^3-2x^2+3x-1}$$
1

$x^4 \qquad -1$

$3x^3-2x^2+3x$

$1 + \frac{3x^3-2x^2+3x}{x^4-1}$ $\quad \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

Wrong!!

Not necessary. this denom yields $\tan^{-1}$ obviously

24
-8
16

$3x^3-2x^2+3x = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+1)$

$x=1 \quad 4 = 4A \Rightarrow A = 1$

$x=-1 \quad -8 = -4B \Rightarrow B = 2 \qquad -1 = D$

$x=0 \quad 0 = A - B - D, \; 0 = 1-2-D \; D = -1$

$x=2 \quad 22 = 15 + 10 + (2C-1)(3)$

$-3 = 3(2C-1)$

$+1 = 2C-1 \Rightarrow C = 0$

$1 + \frac{1}{x-1} + \frac{2}{x+1} - \frac{1}{x^2+1}$

$\ln|x-1| + 2\ln|x+1| - \tan^{-1}x$

using Pg 348

Pg 358

(25) a) $\int \sin^m x\,dx$ If m is odd ie $2N+1$ then

$\int \sin^{2N+1}x\,dx = \int \sin^{2N} x \sin x\,dx$ Now

change $\sin^2 x$ to $1-\cos^2 x$ to get

$\int (1-\cos^2 x)^N \sin x\,dx$ let $u = \cos x$ etc.

If m is even then change $\sin^{2N} x$ to $\left(\frac{1-\cos 2x}{2}\right)^N$ this isn't especially easy.

$\int \sec\theta = \ln|\sec\theta - \tan\theta|$

b) $\int \sec^n x\,dx$ If n is even ie $n = 2m+2$ then $\int \sec^{2m} x \sec^2 x\,dx$, then change $\sec^{2m-2}$ to $(\tan^2 x+1)^m$. let $u = \tan x$ to get $\int (u^2+1)^m\,du$

$\int \sec^3\theta =$

If n is odd its a mess, change to $\frac{1}{\cos^n x}$ multiply by $\frac{\cos x}{\cos x}$ then change denom using $1-\sin^2 x$ Now let $u = \sin x$ etc.

c) $\int \sin^m x \cos^n x\,dx$ if M is odd change to $\int \sin^{m-1} x \cos^n x \sin x\,dx$ change $\sin^{m-1} x$ using $1-\cos^2 x$ then let $u = \cos x$

if N is odd change to

$\int \sin^m x \cos^{n-1} x \cos x$; using $1-\sin^2 x$ ~~then $u=\sin x$~~.

more of c ↓↓↓

C) (Cont) if both m and N are even
change both using $\cos^{2K}x = \left[\frac{1}{2}(1+\cos\frac{\theta x}{2})\right]^K$
and $\sin^{2J}x$ to $\left[\frac{1}{2}(1-\cos\frac{\theta}{2})\right]^J$.

D) $\int \sec^m x \tan^N x$ if m is even change $\sec^{m-2}x$ into
tan x then u = tan x

if m is odd and N is odd Pair up
the extra Sec and Tan for du
then change the remaining tan into
Sec.

If m is odd & N is even
change to Sin over Cos, Keep extra
Sin for dx and ~~substitute~~ 1-cos x
for remaining even Sin, (Not
especially easy.)

change $\tan^{2k}x$ to $(\sec^2-1)^2$
use extra tan as part of
dx. u = secx,

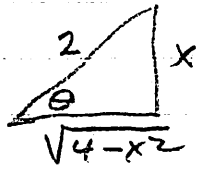
E) $\int \cot^m x \csc^N x \, dx$

if $N \geq 2$, even Keep extra $\csc^2 x$ for dx
change remainder to cot & u = cot x
if M odd, N is even or
If Both m, N are odd Pair up cot Sec for dx
change the rest to sec ✱.

Pg 358

(27) $$\int \frac{\left(\sqrt{4-x^2}\right)^3+1}{\left[(4-x^2)^3+5\right]\sqrt{4-x^2}}\,dx$$

Let $x^2 = 4\sin^2\theta$
$x = 2\sin\theta$
$dx = 2\cos\theta\, d\theta$



$$\int \frac{(8\cos^3\theta+1)\,\cancel{2\cos\theta}\,d\theta}{(64\cos^6\theta+5)\,\cancel{2\cos\theta}}$$

(28) $$\int \frac{x+\sqrt[3]{x-2}}{x^2-\sqrt[3]{x-2}}\,dx$$

Let $u^3 = x-2$
$3u^2du = dx$

$$\int \frac{u^3+2+u\,(3u^2du)}{(u^3+2)^2-u} = \int \frac{3u^5+3u^4+6u^2\,du}{u^6+4u^3-u+4}$$

Pg 351 exmp 3

(29) $$\int \frac{(x^2-5)^7}{x^2+3+\sqrt{x^2-5}}\,dx$$

Let $x = \sqrt{5}\sec\theta$
$dx = \sqrt{5}\sec\theta\tan\theta\, d\theta$
$x^2 = 5\sec^2\theta$

$$\int \frac{5^7\tan^{14}(\sqrt{5}\sec\theta\tan\theta)\,d\theta}{5\sec^2\theta+3+\sqrt{5}\tan\theta}$$

using eg 8 Pg 349

(30) $$\int \frac{\cos^2\theta+\sin\theta}{1-\sin\theta\cos\theta}\,d\theta$$

$$2\int \frac{\left(\frac{1-u^2}{1+u^2}\right)^2+\frac{2u}{1+u^2}}{1-\frac{1-u^2}{1+u^2}\cdot\frac{2u}{1+u^2}}\;\frac{du}{1+u^2} = 2\int \frac{\frac{(1-u^2)^2+2u(1+u^2)}{(1+u^2)^2}}{\frac{(1+u^2)^2-(1-u^2)\,2u}{(1+u^2)^2}}\;\frac{du}{1+u^2}$$

$$2\int \frac{(1-u^2)^2+2u(1+u^2)\,du}{(1+u^2)^3-2u(1+u^2)(1-u^2)} = 2\int \frac{1-2u^2+u^4+2u+2u^3\,du}{1+3u^2+3u^4+u^6+2u-2u^5}$$

$$2\int \frac{u^4+2u^3-2u^2+2u+1\,du}{u^6-2u^5+3u^4+3u^2+2u+1}$$

try again next pg

Pg 358

(33) $\int \frac{\cos x\, dx}{\sin^3 x - 8}$ let $u = \sin x$ $du = \cos x\, dx$

$$\int \frac{du}{u^3-8} = \int \frac{du}{(u-2)(u^2+2u+4)}$$

$$1 = A(u^2+2u+4) + (Bu+C)(u-2)$$

Question
But $u = \sin\theta$
so $u \neq 2$

$u = 2$ $1 = 8A$ $A = \frac{1}{8}$

$u = 0$ $1 = 4A - 2C$ $2 = 1 - 4C$ $C = -\frac{1}{4}$

$u = 1$ $1 = 7A - B + \frac{2}{8}$

$-\frac{1}{8} = -B \Rightarrow B = \frac{1}{8}$

$$\frac{1}{8}\int \frac{du}{u-2} + \int \frac{\frac{1}{8}x - \frac{2}{8}}{u^2+2u+4}$$

$$\frac{1}{8}\int \frac{x-2}{u^2+2u+4}$$

$$\frac{1}{8}\int \frac{du}{u-2} + \frac{1}{8}\int \frac{u+2\, du}{u^2+2u+4} - \frac{4}{8}\int \frac{du}{u^2+2u+4}$$

$$\frac{4}{8}\int \frac{du}{(u+1)^2+3}$$

$$\frac{1}{3}\cdot\frac{4}{8_2}\int \frac{du}{\left(\frac{u+1}{\sqrt{3}}\right)^2+1}$$

$$\frac{1}{8}\ln|u-2| + \frac{1}{8}\ln|u^2+2u+4| - \frac{1}{3}\frac{\sqrt{3}}{1}\tan^{-1}\left(\frac{u+1}{\sqrt{3}}\right)$$

Pg 358

(34) $\int \frac{dx}{\sqrt{2+\sqrt{x}}}$ let $u = \sqrt{x}$
$u^2 = x$
$2u\,du = dx$

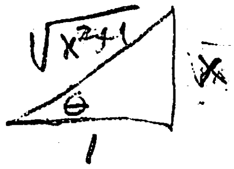
$$\int \frac{2u\,du}{\sqrt{2+u}} = 2\int \frac{u+2-2}{\sqrt{2+u}}\,du =$$

$$2\int \frac{u+2}{\sqrt{u+2}}\,du - 4\int \frac{du}{\sqrt{2+u}} = 2\int \sqrt{u+2}\,du - 4\int \frac{du}{\sqrt{u+2}}$$

$$2\cdot\frac{2}{3}(u+2)^{3/2} - 4\cdot 2\sqrt{u+2} = \frac{4}{3}(u+2)^{3/2} - 8\sqrt{u+2}$$

$$\frac{4}{3}\sqrt{u+2}\left((u+2)-6\right) = \frac{4}{3}\sqrt{u+2}\,(u-4) =$$

$$\frac{4}{3}\sqrt{\sqrt{x}+2}\,(\sqrt{x}-4)$$



(35) $\int \frac{\sqrt{x^2+1}}{x^4}\,dx$ let $x^2 = \tan^2\theta$
$x = \tan\theta$
$dx = \sec^2\theta$

$$\int \frac{\sec\theta\,\sec^2\theta\,d\theta}{\tan^4\theta} = \int \frac{\cos^4\theta}{\sin^4\theta\,\cos^3\theta}\,d\theta$$

$\int \frac{\cos\theta\,d\theta}{\sin^4\theta}$ let $u = \sin\theta$
$du = \cos\theta\,d\theta$

$$\int \frac{du}{u^4} = -\frac{1}{3u^3} = -\frac{1}{3\sin^3\theta} =$$

$$-\frac{1}{3}\,\frac{1}{\frac{x^3}{(x^2+1)^{3/2}}} = -\frac{1}{3}\,\frac{(x^2+1)^{3/2}}{x^3}$$

㊱ $\int \frac{\sin x \, dx}{1+3\cos^2 x}$ $\quad u = \cos x$, $du = -\sin x \, dx$

$-\int \frac{du}{1+3u^2} = -\int \frac{du}{1+(\sqrt{3}u)^2} = -\frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3}u)$

$-\frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3}\cos x)$

㊲ $\int \frac{\sin x}{3+\cos x} dx$ $\quad u = \cos x$, $du = -\sin x$

$-\int \frac{du}{3+u} = -\ln|\cos x + 3|$

Skip ㊳ $\int x\sqrt{x^4-1}\, dx$ $\quad x^2 = \sec\theta$, $2x\,dx = \sec\theta \tan\theta \, d\theta$

$\frac{1}{2}\int \tan^2\theta \sec\theta \, d\theta = \frac{1}{2}\int (\sec^2\theta - 1)\sec\theta \, d\theta$

$u = \tan\frac{\theta}{2}$
$p\, 349$

$\frac{1}{2}\int \frac{\sin^2\theta}{\cos^3\theta} d\theta = \cancel{\frac{1}{2}}\int \frac{4u^2}{\cancel{(1+u^2)^2}} \frac{\cancel{(1+u^2)^3}}{(1-u^2)^3} \cdot \frac{\cancel{2}\,du}{\cancel{1+u^2}}$

$4\int \frac{u^2 du}{(1-u^2)^3} = 4u^2 =$

p358

(40) $\int \sin\sqrt{x}\,dx$ let $z^2 = x$

$2z\,dz = dx$

$2\int z \sin z \, dz$

$u = z$ $v = -\cos z$

$du = dz$ $dv = \sin z \, dz$

$-2z\cos z + 2\int \cos z \, dz$

$-2z\cos z + 2\sin z = -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x}$

$\int \frac{dx}{(4+x^2)^2}$ $\quad \frac{1}{(4+x^2)^2} = \frac{ax+b}{4+x^2} + \frac{cx+d}{(4+x^2)^2}$ #41

$1 = (ax+b)(4+x^2) + cx+d$

$x=0 \quad 1 = 4b + d$

$x=1 \quad 1 = 5(a+b) + c+d$

$x=-1 \quad 1 = 5(-a+b) - c+d$

$2 = 10b + 2d$

$1 = 5b + d$

$1 = 4b + d$

$0 = b \therefore d = 1$

$x=2 \quad 1 = 8(2a+b) + 2c+d$

$1 = 16a + 2c + 1$

$0 = 16a + 2c$

$0 = 8a + c$

$1 = 5a + c + 1$

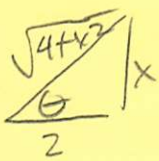
$0 = 5a + c$

$0 = 3a \therefore a = 0$

so $c = 0$

So $\frac{1}{(4+x^2)^2} = \frac{1}{(4+x^2)^2}$ IT is its own partial fraction!

$\int \frac{dx}{(4+x^2)^2} = \int \frac{dx}{(\sqrt{4+x^2})^4}$



Let $\frac{x}{2} = \tan\theta$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\frac{\sqrt{4+x^2}}{2} = \sec\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\int \frac{2\sec^2\theta\, d\theta}{(2\sec\theta)^4}$

$\frac{2}{16}\int \frac{d\theta}{\sec^2\theta}$

$\frac{1}{8}\int \cos^2\theta\, d\theta$

$\frac{1}{8}\cdot\frac{1}{2}\int 1 + \cos 2\theta\, d\theta$

$\frac{1}{16}\left[\theta + \frac{1}{2}\sin 2\theta\right]$

$\frac{1}{16}\left[\tan^{-1}\frac{x}{2} + \cancel{\frac{1}{2}}\cdot\cancel{2}\frac{x}{\sqrt{4+x^2}}\cdot\frac{2}{\sqrt{4+x^2}}\right]$

$\frac{1}{16}\tan^{-1}\frac{x}{2} + \frac{x}{8(4+x^2)}$

Pg 359

(43) $\int \sin^2 3x \cos^2 3x \, dx = \int (\sin 3x \cos 3x)^2 dx =$

$1 \int \left(\frac{1}{2}\sin 6x\right)^2 dx = \frac{1}{4}\int \sin^2 6x \, dx$ let $6x = u$

$6\,dx = du$

formulas #44

$\frac{1}{24}\int \sin^2 u \, du = \frac{1}{24}\left[\frac{u}{2} - \frac{\sin u}{4}\right] =$

$\frac{1}{24}\left[3x - \frac{\sin 12x}{4}\right]$

(44) $\int \sin^3 3x \cos^2 3x \, dx = \int \sin 3x \sin^2 3x \cos^2 3x \, dx =$

$\int (1 - \cos^2 3x) \cos^2 3x \sin 3x \, dx =$

$\int \cos^2 3x \sin 3x \, dx - \int \cos^4 3x \sin 3x \, dx$

$u = \cos 3x \quad du = -3\sin 3x \, dx$

$-\frac{1}{3}\int u^2 du + \frac{1}{3}\int u^4 du = -\frac{1}{9}u^3 + \frac{1}{15}u^5 =$

$-\frac{1}{9}\cos^3 3x + \frac{1}{15}\cos^5 3x = \frac{1}{3}\cos^3 3x \left(\frac{1}{5}\cos^2 3x - \frac{1}{3}\right)$

P_g 358

(46) $\int \frac{x^2\,dx}{x^4-1}$ $\qquad \frac{x^2}{x^4-1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

$$x^2 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$$

$x = 0 \qquad 0 = A - B - D \qquad 0 = \frac{1}{4} + \frac{1}{4} - D$

$x = 1 \qquad 1 = 4A \qquad A = \frac{1}{4}$

$x = -1 \qquad 1 = -4B \qquad B = -\frac{1}{4} \qquad D = \frac{1}{2}$

$x = 2 \qquad 4 = 15A + 5B + 3(2C + \frac{1}{2})$

$4 = \frac{15}{4} - \frac{5}{4} + 6C + \frac{3}{2} \implies C = 0$

$\frac{10}{4}$

$\frac{5}{2}$ 8

$$\int \frac{x^2\,dx}{x^4-1} = \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{1}{4} \ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1} x$$

(47) $\int \frac{x^4+x^2+1}{x^3}\,dx$

$$\begin{array}{r} x \\ x^3 \,\overline{)\,x^4+x^2+1} \\ \underline{x^4 \qquad\quad} \\ x^2+1 \end{array}$$

$\int x\,dx + \int \frac{x^2+1}{x^3}\,dx$ $\qquad$ Let $u = x^3$

$\qquad du = 3x^2\,dx$

$\frac{1}{2}x^2 + \frac{1}{3}\int \frac{du}{u} + \int \frac{dx}{x^3}$

$\frac{1}{2}x^2 + \frac{1}{3}\ln|u| - \frac{1}{2}x^{-2} = \frac{1}{2}x^2 + \frac{1}{3}\ln x^3 - \frac{1}{2x^2} =$

$\frac{1}{2}x^2 + \ln|x| - \frac{1}{2x^2}$

Pg 358

㊾ $\int 10^x dx$ $10 = e^{\ln 10}$

$10^x = e^{x \ln 10}$

$$\int e^{x \ln 10} dx = \frac{1}{\ln 10} e^{x \ln 10} = \frac{10^x}{\ln 10}$$

(53) $\int \cos^2 x \, dx = \frac{1}{2}\int 1 + \cos 2x \, dx =$

$\frac{x}{2} + \frac{1}{4} \sin 2x$

Pg 359

used as Quiz pblm

(76) $\int x^3 \tan^{-1} x \, dx$

$u = \tan^{-1} x \quad v = \frac{1}{4} x^4$

$du = \frac{1}{x^2+1} dx \quad dv = x^3 dx$

$x^4 \tan^{-1} x - \frac{1}{4} \int \frac{x^4}{x^2+1} dx$

$$\begin{array}{r} x^2-1 \\ x^2+1\overline{)\,x^4} \\ x^4+x^2 \\ \hline -x^2 \\ -x^2-1 \\ \hline 1 \end{array}$$

$x^4 \tan^{-1} x - \frac{1}{4} \int x^2 - 1 \, dx + \int \frac{dx}{x^2+1}$

$x^4 \tan^{-1} x - \frac{1}{12} x^3 + \frac{1}{4} x + \tan^{-1} x$

$(x^4+1) \tan^{-1} x - \frac{1}{12} x^3 + \frac{1}{4} x$

? (78) $\int \frac{dx}{x^3+4x} = \int \frac{dx}{x(x^2+4)}$ $\quad \frac{1}{x(x^2+4} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$1 = A(x^2+4) + Bx$

$x = 0 \quad 1 = 4A \Rightarrow A = \frac{1}{4}$

$x = 1 \quad 1 = \underset{\frac{5}{4}}{5A} + B$

$-\frac{1}{4} = B$

$\frac{1}{4} \int \frac{dx}{x} - \frac{1}{4} \int \frac{dx}{}$

Pg 360

(148) $\int \frac{x^3\,dx}{(x-1)^2}$

$$\begin{array}{r} x+2 \\ x^2-2x+1 \overline{) x^3} \\ \underline{x^3-2x^2+x} \\ 2x^2-x \\ \underline{2x^2-4x+2} \\ 3x-2 \end{array}$$

A)

$$\int x+2\,dx + \int \frac{3x-2}{(x-1)^2}$$

$$\frac{3x-2}{(x-1)^2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2}$$

$$3x-2 = A(x-1) + B$$

$x=0 \quad -2 = -A + B$

$x=1 \quad 1 = B \quad \therefore A = 3$

$$\int x+2\,dx + 3\int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

$$\frac{1}{2}x^2 + 2x + 3\ln|x-1| - \frac{1}{x-1}$$

B) $\int \frac{x^3\,dx}{(x-1)^2}$ let $u = x-1$, $du = dx$, $x = u+1$ $\quad \int \frac{(u+1)^3\,du}{u^2} =$

$$\int \frac{u^3+3u^2+3u+1}{u^2}\,du = \int u + 3 + \frac{3}{u} + \frac{1}{u^2}\,du$$

$$\frac{1}{2}u^2 + 3u + 3\ln|u| - \frac{1}{u^2} =$$

$$\frac{1}{2}(x-1)^2 + 3(x-1) + 3\ln|x-1| - \frac{1}{(x-1)^2}$$

$\frac{1}{2}x^2 - x + \frac{1}{2} + 3x - 3$ etc.

this differs by $-\frac{5}{2}$

Pg 360

(151) $\int \frac{x^3}{\sqrt{1+x^2}}\,dx$ let $x^2 = \tan^2\theta$

$x = \tan\theta$

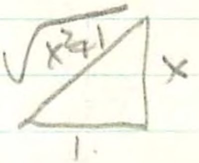
$dx = \sec^2\theta\,d\theta$

$$\int \frac{\tan^3\theta \sec^2\theta\,d\theta}{\sec\theta} = \int \tan^3\theta \sec\theta\,d\theta =$$

$$\int \tan^2\theta \tan\theta \sec\theta\,d\theta = \int (\sec^2\theta - 1)\tan\theta\sec\theta\,d\theta$$

$$\int \sec^2\theta \tan\theta \sec\theta\,d\theta - \int \tan\theta\sec\theta\,d\theta$$

$$\frac{1}{3}\sec^3\theta - \sec\theta$$



$$\frac{1}{3}\left(\sqrt{x^2+1}\right)^3 - \sqrt{x^2+1}$$

I guess I used the tables to keep The Book open to pg 360.

(152) $\int x\sqrt[3]{x+1}\,dx$ let $u = \sqrt[3]{x+1}$

$u^3 = x+1$

$3u^2du = dx$

A)

$$3\int (u^3-1)u^3\,du = 3\int u^6 - u^3\,du =$$

$$\frac{3}{7}u^7 - \frac{3}{4}u^4 = \frac{3}{7}(x+1)^{7/3} - \frac{3}{4}(x+1)^{4/3}$$

B) let $u = x+1$ $\int (u-1)(u)^{1/3}du = \int u^{4/3} - u^{1/3}du$

$du = dx$

$$\frac{3}{7}u^{7/3} - \frac{3}{4}u^{\frac{4}{3}} = \frac{3}{7}(x+1)^{7/3} - \frac{3}{4}(x+1)^{4/3}$$

$$\int \frac{x^2}{\sqrt{1+x}}\,dx$$

Pg 360

(153) let $u = \sqrt{1+x}$

$u^2 = 1+x$

A) $2u\,du = dx$

$$2\int \frac{(u^2-1)^2 \cancel{u}\,du}{\cancel{u}} =$$

$$2\int u^4 - 2u^2 + 1\,du = \frac{2}{5}u^5 - \frac{4}{3}u^3 + 2u =$$

$$\frac{2}{5}(1+x)^{5/2} - \frac{4}{3}(1+x)^{3/2} + 2\sqrt{1+x}$$

B) let $u = 1+x$

$du = dx$

$$\int \frac{(u+1)^2}{u^{\frac{1}{2}}}\,du = \int \frac{u^2+2u+1}{u^{\frac{1}{2}}}\,du$$

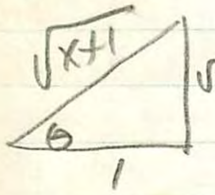
$$\int u^{3/2} + 2u^{\frac{1}{2}} + \frac{1}{u^{\frac{1}{2}}}\,du = \frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} + 2u^{\frac{1}{2}}$$

which is the same as a,

c) let $x = \tan^2\theta$

$dx = 2\tan\theta\sec^2\theta\,d\theta$

$$2\int \frac{\tan^5\theta\sec^2\theta}{\sec\theta}\,d\theta =$$



$$2\int \tan^5\theta\sec\theta\,d\theta = 2\int \tan^4\theta\tan\theta\sec\theta\,d\theta =$$

$$2\int (\sec^2\theta - 1)^2\tan\theta\sec\theta\,d\theta =$$

$$2\int (\sec^4\theta - 2\sec^2\theta + 1)\tan\theta\sec\theta\,d\theta$$

$$\frac{2}{5}\sec^5\theta - \frac{4}{3}\sec^3\theta + 2\sec\theta$$

$$\frac{2}{5}(x+1)^{5/2} - \frac{4}{3}(x+1)^{3/2} + 2\sqrt{x+1}$$

(154) · $u(\tau)$ = rate of flow at time τ

$U(\tau) = \int_0^{\tau} u(s)\,dx$ is Amt of water which flows during interval $0 \to \tau$.

$Z(\tau) =$ rate of Demand for. power. at time τ,

$Z(s) - u(s)$ is Difference of Demand and availibility. if Demand is larger (ie $Z(s) > u(s)$) then we will need additional energy.

if rate of cost of additional energy is $C(s)$ then

$C(Z(s)-u(s))$ is cost of that additional energy at time s,

$\int_0^{\tau} C(Z(s)-u(s))\,ds$ = total cost of outside energy.

Given $\int \frac{e^x}{x} dx$ non elementary.

WTP $\int \frac{1}{\ln x} dx$ is also non elementary.

$\int \frac{1}{\ln x} dx$

let $u = \ln x$

$e^u = x$

$e^u du = dx$

$\int \frac{1}{u} \cdot e^u du =$

$\int \frac{e^u}{u} du$ not elementary.